Review of Particle Physics

Here I follow D. Griffiths, Introduction to Elementary Particles and Particle Data Group: book & booklet

This is not a course in Rel. Quant. Mech. Or QFT. Or particles. We will cover a lot of material shallowly.

A particle is a quantum state. It may, or it may not, be characterized by quantum numbers. Example:

$|p\rangle$ is a proton, $|n\rangle$ is a neutron; $\alpha|p\rangle + \beta|n\rangle$ is also a particle.

Elementary particles are the irreducible components of matter. The existence and interaction of composite particles (e.g. bound states) is a consequence of the interaction of elementary particles.

Elementary particles and the 4 forces

FERMIONS matter constituents spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e electron	0.000511	–1
ν_M middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ muon	0.106	–1
ν_H heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ tau	1.777	–1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.002	2/3
d down	0.005	–1/3
c charm	1.3	2/3
s strange	0.1	–1/3
t top	173	2/3
b bottom	4.2	–1/3

BOSONS force carriers spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin =1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
Strength at 3×10^{-17} m	10^{-41}	10^{-4}	1	60

http://www.cpepphysics.org/particle-chart.html

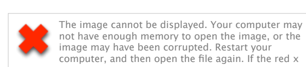
+ Higgs

Hadrons: composite particles

Mesons $q\bar{q}$ **Mesons are bosonic hadrons** **These are a few of the many types of mesons.**					
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
π^+	pion	$u\bar{d}$	+1	0.140	0
K^-	kaon	$s\bar{u}$	–1	0.494	0
ρ^+	rho	$u\bar{d}$	+1	0.776	1
B^0	B-zero	$d\bar{b}$	0	5.279	0
η_c	eta-c	$c\bar{c}$	0	2.980	0

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$ **Baryons are fermionic hadrons.** **These are a few of the many types of baryons.**					
Symbol	Name	Quark content	Electric charge	Mass GeV/c^2	Spin
p	proton	uud	1	0.938	1/2
$\bar{p}$	antiproton	$\bar{u}\bar{u}\bar{d}$	–1	0.938	1/2
n	neutron	udd	0	0.940	1/2
Λ	lambda	uds	0	1.116	1/2
Ω^-	omega	sss	–1	1.672	3/2

http://www.cpepphysics.org/particle-chart.html

All states of the form 

are allowed.

Tetraquarks, $q\bar{q}q\bar{q}$, and pentaquarks, $qqqq\bar{q}$, have been observed.

N.B. The interpretation of tetra-quarks/penta-quarks is under debate. Tetraquarks are less certain.

Decays

Knowledge of particle physics is due to observation of <u>decays</u>, <u>scattering</u> and <u>bound states</u> (mesons and baryons).

For an unstable particle, Γ is the probability of decay per unit of time. Thus: $dN = -\Gamma N dt$

$$N(t) = N(0)e^{-t\Gamma} \qquad \tau = 1/\Gamma$$

For multiple decay channels, e.g.

$\tau^+ \to e^+ + \nu_e + \bar{\nu}_\tau$ B=0.1736

$\tau^+ \to \mu^+ + \nu_\mu + \bar{\nu}_\tau$ B=0.1785 (pdg 2010)

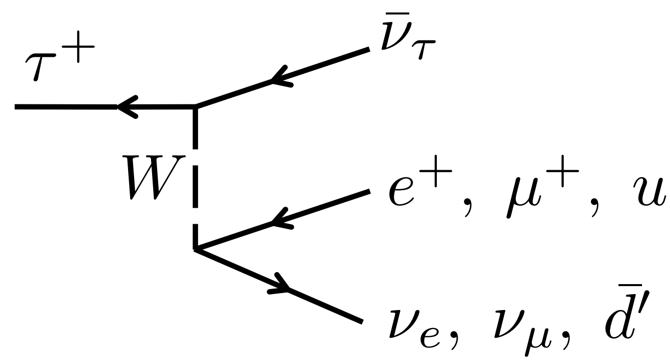
$\tau^+ \to \bar{\nu}_\tau + mesons$ B=0.6304

(a few rare modes not listed)

$$\Gamma_{Tot} = \sum \Gamma_i \qquad B = \Gamma_i/\Gamma_{Tot}$$

Decays – Example: evidence for color

Tau decay



$m_\tau >> m_e,\ m_\mu,\ m_u,\ m_d$: we can use Fermi's Golden rule

Each decay mode is equally likely, but the $u\bar{d}'$ final state counts for 3 (color), so the naïvely predicted branching ratios are 0.2, 0.2 and 0.6

$\tau^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\tau$ B=0.1736

$\tau^+ \rightarrow \mu^+ + \nu_\mu + \bar{\nu}_\tau$ B=0.1785

$\tau^+ \rightarrow \bar{\nu}_\tau + mesons$ B=0.6304

(a few rare modes not listed)

Relative strength of forces

Strong decays. Typical hadron range $10^{-22} - 10^{-24}$ s

$\Delta^{++} \rightarrow p + \pi^{+}$ B = 100% $\tau = 5.6\text{x}10^{-24}$ s

Electromagnetic decays. Typical hadron range $10^{-16} - 10^{-21}$ s

$\pi^{0} \rightarrow \gamma + \gamma$ B = 98.8% $\tau = 8.4\text{x}10\text{-}^{17}$ s

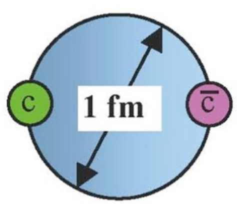
Weak decays. Typical hadron range $10^{-7} - 10^{-13}$ s

$\pi^{+} \rightarrow \mu^{+} + \nu_{\mu}$ B = 99.99% $\tau = 2.6\text{x}10^{-8}$ s

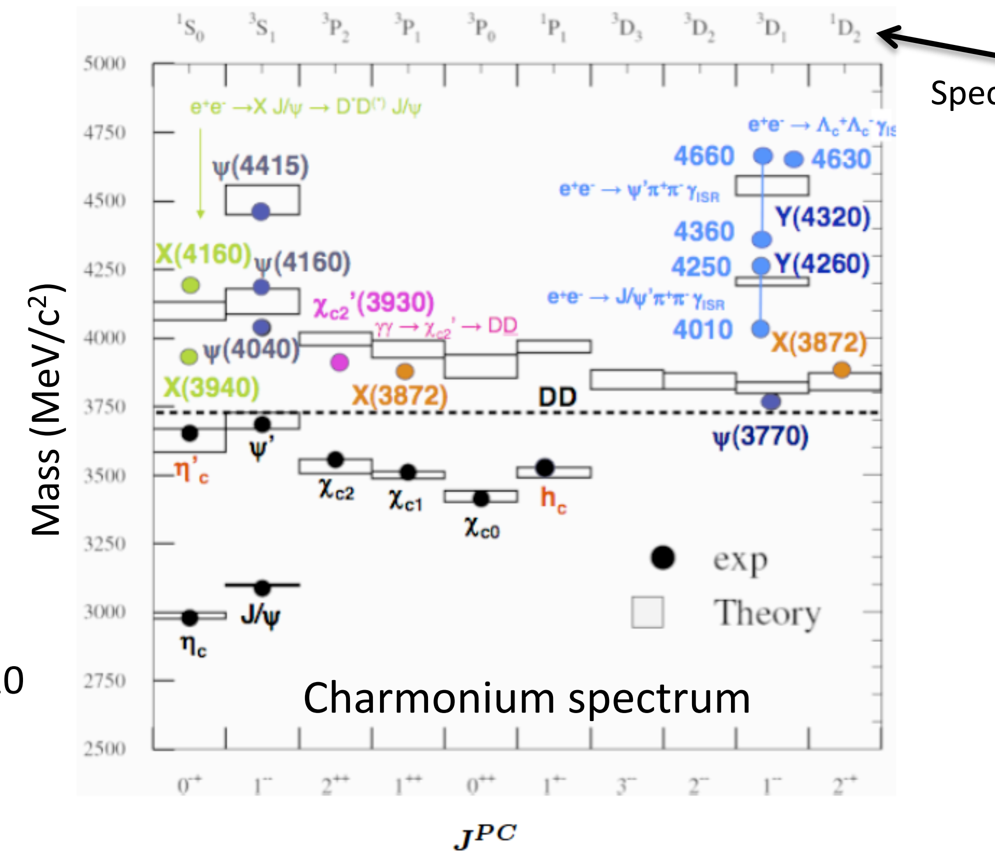
$\Lambda \rightarrow p + \pi^{-}$ B = 63.9% $\tau = 2.6\text{x}10^{-10}$ s

Bound states

Charmonium ($c\bar{c}$) and Bottomium ($b\bar{b}$) are bound states of quarks that are massive enough to be non-relativistic. The mass spectrum is reasonably well described using Schödinger's equation (e.g. fit potential $a\ r + b/r$). Modern methods use lattice QCD. Strong potential is found in the 10^{-15} m range. The "line" splitting are of similar origin to the fine and hyperfine structure in the hydrogen atom.



T. Barnes, Proc. Charm 2010
IJMPCS 2 (2011) 214-223

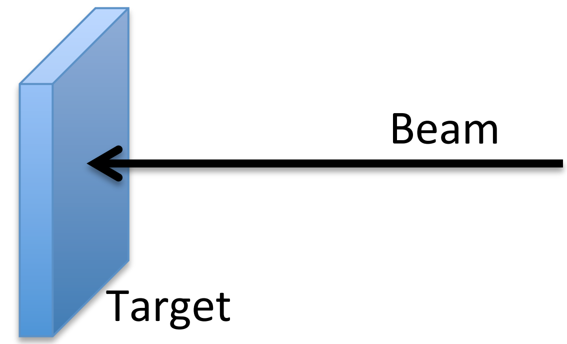


Spectroscopic State

Cross Section & Scattering

Scattering is the effective size of a target. For a classical contact force the cross section is equal to the geometrical cross section.

For a beam flux J (part/area.time) and illuminated targets N, the interaction rate is $R_i = JN\sigma_i = L\sigma_i$, where L is known as the luminosity (1/area.time)



Given a beam and target, there can be multiple possible outputs:

$$\begin{aligned} e + p &\to e + p \\ &\to e + p + \gamma \\ &\to e + p + \pi^0 \end{aligned}$$

The total cross section is

$$\sigma_{Tot} = \sum_i \sigma_i$$

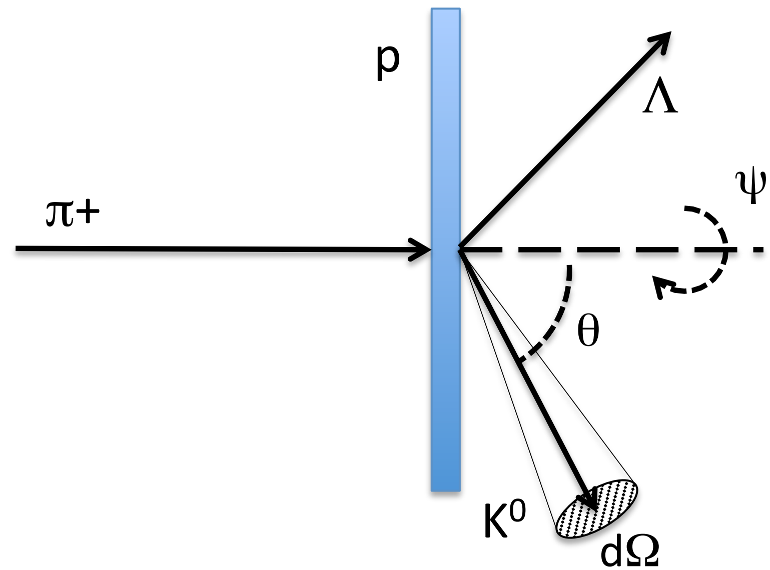
Differential cross section

Two particles -> Two particles

$$\pi^+ + p \to K^0 + \Lambda$$

The angular distribution of the kaons is given by the differential cross section $\frac{d\sigma}{d\Omega}$. So that the interaction rate per solid angle for channel i, is

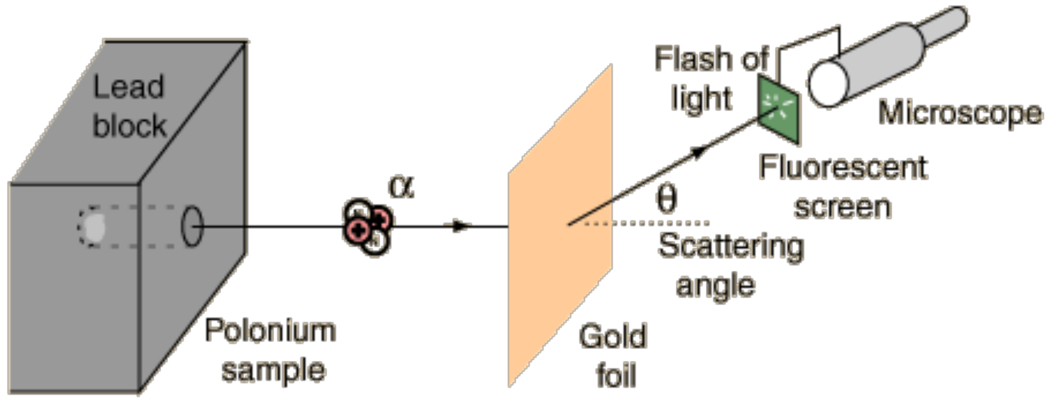
$$\frac{dR_i}{d\Omega} = JN\frac{d\sigma_i}{d\Omega}$$



θ: scattering angle

ψ: azimuthal angle

Scattering – Example: Rutherford scattering



wikipedia

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2 Z_1 Z_2}{8\pi\epsilon_0 E}\right)^2 \csc^4(\theta/2)$$

E: beam kinetic energy

Internal structure (e.g. for nuclei) would manifest itself in deviations from Rutherford's formula. Lack of known deviations from point-like structure sets a limit on the radius of the electron.

Relativistic Quantum Mechanics

You arrive at Schrödinger´s equation by quantizing $\frac{|\vec{p}|^2}{2m} + V = E$ with

$$\vec{p} \to -i\hbar\nabla \qquad E \to i\hbar\frac{\partial}{\partial t}$$

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial}{\partial t}\Psi$$

Schrödinger equation

The Klein-Gordon equation can be obtained with the same prescription starting with: (Let's ignore the potential)

$$p^2 - m^2c^2 = E^2/c^2$$

$$p_\mu \to i\hbar\frac{\partial}{\partial x_\mu} = i\hbar\partial_\mu$$

$$-\hbar^2\partial^\mu\partial_\mu\Psi - m^2c^2\Psi = 0$$

$$-\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} + \nabla^2\Psi = \left(\frac{mc}{\hbar}\right)^2\Psi$$

Klein-Gordon equation

Relativistic Quantum Mechanics

The Klein-Gordon equation has at least two problems:

1. It has negative energy solutions
2. $|\Psi(\vec{r})|^2$ does not have a probabilistic interpretations

The first problem is related to the quadratic nature of in the energy: $p^2 - m^2c^2 = 0$. The second problem is solved in QFT. Though Klein-Gordon by itself is not a good Quantum theory, within QFT it properly represents spin 0 particles.

Dirac set out to find an equation consistent with $p^2 - m^2c^2 = 0$ but linear in energy once quantized. In a sense you want to take the square root of an operator so you are consistent with:

$$E = +\sqrt{|\vec{p}|^2c^2 + m^2c^4}$$

Relativistic Quantum Mechanics

Dirac proposed an equation linear in the partial derivatives

$$i\hbar\frac{\partial\Psi}{\partial t} = (-i\hbar c\sum_i^3 \gamma^i\frac{\partial}{\partial x_i} + \gamma^0 mc^2)\Psi$$

$$i\hbar\gamma^\mu\partial_\mu\Psi - mc\Psi = 0$$

Consistency with $p^2 - m^2c^2 = 0$ implies $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$

So γ^μ are not complex numbers. The following set of 4x4 matrices work (but this is not a unique choice)

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

Here 0 denotes a 2x2 null matrix, 1 denotes the 2x2 identity matrix and σ^i denotes the Pauli matrices. The wavefunction has 4 elements:

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \end{pmatrix}$$

Relativistic Quantum Mechanics

Let's look for $\vec{p}=0$ or $\partial\Psi/\partial x = \partial\Psi/\partial y = \partial\Psi/\partial z = 0$ solutions to Dirac's equation.

$$\frac{i\hbar}{c}\gamma^0\frac{\partial\Psi}{\partial t} - mc\Psi = 0$$

$$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\partial\Psi_A/\partial t\\ \partial\Psi_B/\partial t\end{pmatrix} = -i\frac{mc^2}{\hbar}\begin{pmatrix}\Psi_A\\ \Psi_B\end{pmatrix}$$

Where

$$\Psi_A = \begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} \qquad \Psi_B = \begin{pmatrix}\Psi_3\\ \Psi_4\end{pmatrix}$$

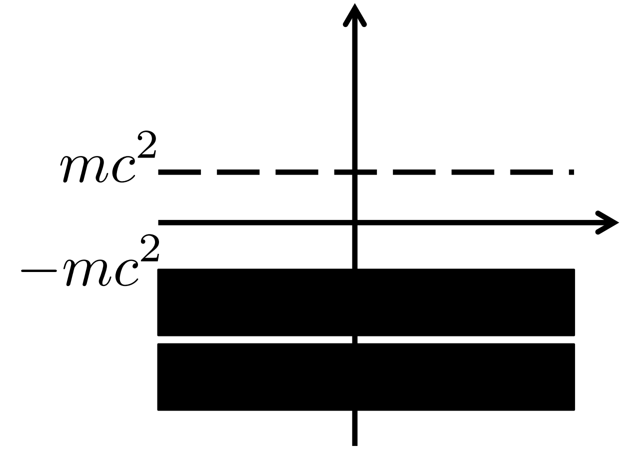
The solutions are

$$\Psi_A(t) = e^{-imc^2t/\hbar}\Psi_A(0) \qquad \Psi_B(t) = e^{imc^2t/\hbar}\Psi_B(0)$$

Good: spin ½ appears naturally. Probabilistic interpretation of the wavefunction is restored.
Bad: negative energy solutions are still here.

Particles & antiparticles

Dirac created a vacuum in which all negative energy states are full. A positron is a negative energy state that is not full.



A Positron

Non-relativistic QM and Relativistic QM describe single particles states. QFT describes many particle states (including vacuum).

Within QFT the negative energy solutions are interpreted as antiparticles. There's no need for a 'sea' of filled states to represent vacuum. Probabilistic interpretation works just fine. QFT can accommodate bosons (e.g. spin 0 or 1) or fermions (e.g. spin ½).

Feynman diagrams are used to calculate amplitudes of approximation theory (scattering or decays).

Symmetries and conserved numbers

Noether Theorem: Symmetry <-> Conservation law

Translation in Time <-> Energy
Translation in Space <-> Momentum
Rotations <-> Angular Momentum

In an isolated system, Energy, momentum and angular momentum are always conserved. There are other symmetries we will study. Some are perfect symmetries conserved by all forces (baryon number), some are perfect symmetries, conserved by some forces (parity), some are imperfect symmetries (isospin)

Example – Rotations

Assume a Hamiltonian invariant under rotations, e.g.

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$
$$z' = z$$

$$H(\vec{r}\,') = H(\vec{r})$$

The infinitesimal transformation is

$$x' = x - y\delta\theta$$
$$y' = x\delta\theta + y$$
$$z' = z$$

Let $\hat{R}_z$ be the operator for an infinitesimal rotation about z

$$\hat{R}_z\Psi(\vec{r}) = \Psi(\vec{r}\,') = \Psi(x - y\delta\theta, x\delta\theta + y, z)$$
$$= \Psi(\vec{r}) - \delta\theta(y\frac{\partial}{\partial x} - x\frac{\partial}{\partial y})\Psi(\vec{r})$$

Example – Rotations

$$\hat{R}_z \Psi(\vec{r}) = \Psi(\vec{r}\,') = (1 + i\frac{\delta\theta}{\hbar} L_z)\Psi(\vec{r})$$

Let's find the commutator of the Hamiltonian with the rotation operator

$$[\hat{R}_z, H]\Psi(\vec{r}) = (R_z H - H R_z)\Psi(\vec{r}) = H(\vec{r}\,')\Psi(\vec{r}\,') - H(\vec{r})\Psi(\vec{r}\,')$$

Since $H(\vec{r}') = H(\vec{r})$ then $[R_z, H]\Psi(\vec{r}) = 0$. And because the waveform chosen to evaluate this is arbitrary, then

$$[\hat{R}_z, H] = 0$$

$$[L_z, H] = 0$$

I will not go on reviewing the properties of angular momentum in Quantum Mechanics. I'll assume you know them.

Spectroscopic classification of hadrons

Mesons are $q\bar{q}$ states. There's an orbital angular momentum quantum number and you have to consider the spin of the quarks. You can classify the states by the orbital number and by the summed spin of the quark-antiquark. The sum of this spin and orbital number give you the meson spin.

Here 'sum' means addition of angular momentum in quantum mechanics.

The spectroscopic notation is often used

$$^{2s+1}L_j$$

The same is true for baryons (qqq) but you now have 2 orbital numbers that are 'added' up to L. Also the total spin combinations are for 3 spin ½ particles

Spectroscopic notation of hadrons – Examples

Mesons of the type $u\bar{d}$

$$\pi^+ : \quad {}^1S_0$$

$$\rho^+(770) \quad {}^3S_1$$

Baryons of the type uud

$$p : \quad {}^2S_{1/2}$$

$$\Delta^+ : \quad {}^4S_{3/2}$$

Mesons of the type $c\bar{c}$

$$\eta_c : \quad {}^1S_0$$

$$J/\Psi : \quad {}^3S_1$$

$$\chi_{c0} : \quad {}^3P_0$$

$$\chi_{c2} : \quad {}^3P_2$$

$$\chi_{c1} : \quad {}^3P_1$$

Intrinsic Parity

Parity is conserved by QED & QCD but not by weak interactions.

Assume a Hamiltonian that is invariant under $\vec{r} \to \vec{r}\,'$

Let the parity operator $\hat{P}$, be the the operator that does this transformation $\Psi(-\vec{r}) = \hat{P}\Psi(\vec{r})$

It's clear that $\hat{P}^2 = 1$, which implies that parity has eigenvalues ±1.

Let's look at the effect of parity on plane wave

$$\hat{P}\Psi_{\vec{p}}(\vec{r},t) = \hat{P}e^{i(\vec{p}\cdot\vec{r}-Et/\hbar)} = P_a e^{i(-\vec{p}\cdot\vec{r}-Et/\hbar)} = P_a \Psi_{-\vec{p}}(\vec{r},t)$$

For $\vec{p} = 0$, $\Psi_{\vec{p}=0}$ is an eigenstate with eigenvalue P_a. This is the intrinsic parity of a particle.

Using Dirac's equation it can be shown that for a fermion/antifermion

$$P_f = -P_{\bar{f}}$$

It is customary to choose

$$P_{e^-} = P_{\mu^-} = P_{\tau^-} = P_{\text{quark}} = 1$$

$$P_{e^+} = P_{\mu^+} = P_{\tau^+} = P_{\text{anti-quark}} = -1$$

Also unproven: The photon parity is negative $P_\gamma = -1$

Parity of bound states

Orbital angular momentum also has well defined parity

$$\Psi(\vec{r},t) = R_{nl}(r)Y_l^m(\theta,\phi)$$

Applying parity $\vec{r} \to \vec{r}\,'$ $\quad r \to r$

$$\theta \to \pi - \theta$$

$$\phi \to \phi + \theta$$

$$R_{nl}(r) \to R_{nl}(r) \qquad Y_m^l(\theta,\phi) \to (-1)^l Y_m^l(\theta,\phi)$$

The parity of a composite system (e.g. a meson) is the product of the intrinsic parity of the components and the term from angular momentum. $P_{\mathrm{meson}} = P_q P_{\bar{q}}(-1)^l$

$$\pi^+ : \quad {}^1S_0 \qquad P_{\pi^+} = P_u P_{\bar{d}}(-1)^0 = -1$$

$$\rho^+(770) \quad {}^3S_1 \qquad P_{\rho^+(770)} = P_u P_{\bar{d}}(-1)^0 = -1$$

It's messier for baryons (two orbital numbers), but the idea is the same

$$P_{\mathrm{baryon}} = P_{\mathrm{q}}^3(-1)^{l_{12}}(-1)^{l_3}$$

Example of Ang. Momentum / Parity conservation

The process $\eta \to \pi^0\pi^0\pi^0$ has been seen. The process $\eta \to \pi^0\pi^0$ has not been seen. Note that $\eta \to 2\gamma$ dominates: EM decay

The parity of the 3 pions has to be the same as for the eta.

$$P_\eta = P^3_{\pi^0}(-1)^{l_{12}}(-1)^{l_3}$$

The eta and pion have spin 0. Angular momentum conservation implies $l_{12} = l_3$

Which implies $P_\eta = -1$

Angular momentum conservation: $l = 0$

Now $P_{\pi^0\pi^0} = P^2_{\pi^0}(-1)^l = 1$

The decay of eta into two pions does not conserve parity.
By the way eta is ${}^1S_0$ and has quark content $\alpha u\bar{u} + \beta d\bar{d} + \gamma s\bar{s}$

Lepton number, flavor numbers, baryon number

Lepton number is conserved universally (*)

$$L_e = N(e^-) - N(e^+) + N(\nu_e) - N(\bar{\nu}_e)$$

$$L_\mu = N(\mu^-) - N(\mu^+) + N(\nu_\mu) - N(\bar{\nu}_\mu)$$

$$L_\tau = N(\tau^-) - N(\tau^+) + N(\nu_\tau) - N(\bar{\nu}_\tau)$$

Quark flavor numbers are conserved by QED, QCD but violated by weak interactions

$$S = N(\bar{s}) - N(s)$$

$$C = N(c) - N(\bar{c})$$

$$\tilde{B} = N(\bar{b}) - N(b)$$

$$T = N(t) - N(\bar{t})$$

Quark numbers for u & d are conserved, but they are linear combination of these 4 numbers, charge & baryon number

Baryon number is universally conserved

$$B = \frac{1}{3}(N(q) - N(\bar{q}))$$

Examples

$$\pi^- \to \pi^0 + e^- + \bar{\nu}_e$$
$$d\bar{u} \to (u\bar{u}, d\bar{d}) + e^- + \bar{\nu}_e$$

This violates quark numbers for u,d
Only possible as a weak decay

$$\gamma + p \to \pi^+ + n$$
$$\gamma + (uud) \to u\bar{d} + (udd)$$

Everything conserved. Since there are photons, it's an EM interaction.

$$p + \bar{p} \to \pi^+ + \pi^- + \pi^0$$
$$(uud) + (\bar{u}\bar{u}\bar{d}) \to u\bar{d} + d\bar{u} + (u\bar{u}, d\bar{d})$$

Everything conserved. This is a strong process

$$p + \pi^- \to K^0 + \Lambda$$
$$(uud) + (\bar{u}d) \to d\bar{s} + (uds)$$

Everything conserved. Strange particles created as pairs. This is a strong process.

$$\Lambda \to p + \pi^-$$
$$(uds) \to (uud) + (\bar{u}d)$$

Strangeness violated. Weak process.

Isospin

The masses of the proton and neutron are very similar (938.28 MeV and 939.57 MeV respectively). Let's think of the proton and neutron as a degenerate two state system. A degenerate 2-state system can be described as a "spin-1/2" system. Hence this internal symmetry is called "Isospin".

$$p = |\frac{1}{2}\ \frac{1}{2}> \qquad n = |\frac{1}{2}\ -\frac{1}{2}>$$

Strong interactions conserve isospin. The group of symmetry is SU(2), just as for angular momentum. Just as for angular momentum there are singlets, doublets, triplets, etc.

Singlet: $\Lambda = |0\ 0>$

Triplet: $\pi^+ = |1\ 1> \quad \pi^0 = |1\ 0> \quad \pi^- = |1\ -1>$

Tetraplet: $\Delta^{++} = |\frac{3}{2}\ \frac{3}{2}> \quad \Delta^+ = |\frac{3}{2}\ \frac{1}{2}> \quad \Delta^0 = |\frac{3}{2}\ -\frac{1}{2}> \quad \Delta^- = |\frac{3}{2}\ -\frac{3}{2}>$

$$m_{\pi^\pm} = 139.6\ \mathrm{MeV} \quad m_{\pi^0} = 135\ \mathrm{MeV} \quad m_\Delta = 1232\ \mathrm{MeV}$$

Isospin

The isospin symmetry arises because the up and down quarks have almost identical mass and can be described as degenerate 2-body system:

$$u = |\frac{1}{2}\ \frac{1}{2} > \qquad d = |\frac{1}{2}\ -\frac{1}{2} >$$

Two quantum numbers characterize isospin I and I_3. This latter can be found thus:

$$I_3 = \frac{1}{2}(N(u) - N(\bar{u}) - N(d) + N(\bar{d}))$$

The very same rules of adding angular momentum apply to isospin. To calculate the isospin of a bound composite particle (hadron) start with the isospin of the components. If a given particle has isospin I, then there are 2I+1 members in that multiplet

Pion-Nucleon scattering

Consider these 6 elastic processes

$$\pi^+ + p \to \pi^+ + p \qquad \pi^+ + n \to \pi^+ + n$$
$$\pi^0 + p \to \pi^0 + p \qquad \pi^0 + n \to \pi^0 + n$$
$$\pi^- + p \to \pi^- + p \qquad \pi^- + n \to \pi^- + n$$

And 4 charge exchange processes

$$\pi^+ + n \to \pi^0 + p \qquad \pi^0 + p \to \pi^+ + n$$
$$\pi^0 + n \to \pi^- + p \qquad \pi^- + p \to \pi^0 + n$$

Strong interactions don't distinguish protons from neutrons or among pions. Since (p,n) have I=1/2 and the pions have I=1, the total isospin on both sides of each process is 1/2 or 3/2.

C-Parity

C-Parity is conserved by QED & QCD but not by weak interactions.

Let $\hat{C}$ be the operator that exchange particle with antiparticle.

$$\hat{C}|a >= |\bar{a} >$$

In principle there's a phase. But it can be shown that it doesn't add physical meaning. As with parity $\hat{C}^2 = 1$, which again implies eigenvalues ± 1.

For particles that are their own antiparticle $(\gamma, \pi^0, \dots)$

$$\hat{C}|b >= C_b|b >$$

Note that many particles $(e^-, p, n, \dots)$ are not C-parity eigenstates.

Left unproven: the C-parity of the photon is negative.

A particle-antiparticle pair is also a C-parity eigenstate.

Example: $|\pi^+, \pi^-; l >$

Recall that the pions have zero spin

$$\hat{C}|\pi^+, \pi^-; l >= (-1)^l|\pi^+, \pi^-; l >$$

C-Parity of particle-antiparticle pairs

Let's study $|e^+, e^-; l = 0 >$

Because the electron is spin ½ it can be in the symmetric triplet or the antisymmetric singlet.

$$\begin{array}{c} |++> \\ \frac{1}{\sqrt{2}}(|+-> + |-+>) \\ |--> \end{array} \qquad \frac{1}{\sqrt{2}}(|+-> - |-+>)$$

$$s_{e^+e^-} = 1 \qquad s_{e^+e^-} = 0$$

So we find:

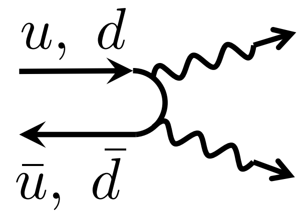
$$\hat{C}|e^+, e^-; l = 0 >= (-1)^{s+1}|e^+, e^-; l = 0 >$$

Putting all together and adding a magic factor of -1 from exchange of fermions in QFT

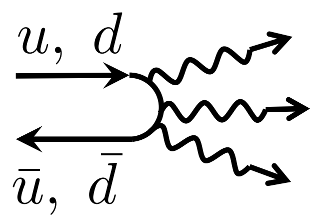
$$\hat{C}|f^+, f^-; l, s >= (-1)^{l+s}|e^+, e^-; l, s >$$ (fermions)

Example of C-parity

The process $\pi^0 \to \gamma\gamma$ has been seen. The Feynman diagram is



Naively one would expect that $\pi^0 \to \gamma\gamma\gamma$ is possible



And

$$\frac{\Gamma(\pi^0 \to 3\gamma)}{\Gamma(\pi^0 \to 2\gamma)} \sim O(\alpha)$$

However

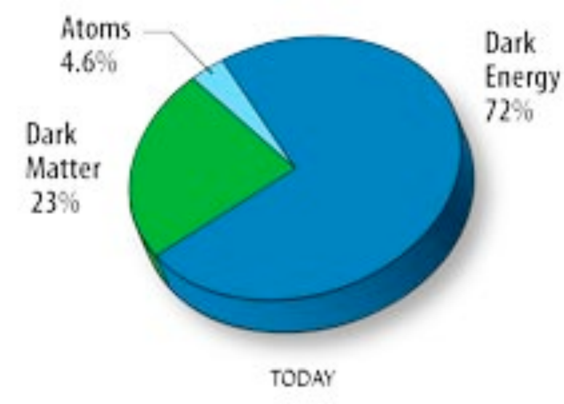
$$C_{\pi^0} = 1 \quad C_\gamma = -1$$

$$C_{2\gamma} = C_\gamma^2 = 1 \quad C_{3\gamma} = C_\gamma^3 = -1$$

Which means the 3 photon decays is forbidden (and it hasn't been observed). By the way π^0 is ${}^1S_0$ and has quark content $\alpha u\bar{u} + \beta d\bar{d}$

Beyond the Standard Model

As you surely know that the baryons in the Universe are matter and that there is little anti-matter. Why isn't there an equal mixture of both? This is, in good measure, a particle physics question.



wikipedia

As you surely know, the fraction of the Universe made by leptons and hadrons (what the Standard Model can explain) is small. The Universe is mostly made of Dark Energy and Dark Matter.

Neither of those can be explained within the Standard Model.

Sakharov Conditions – Matter/Antimatter

The Universe can develop a matter/anti-matter asymmetry if the following are satisfied

- Baryon number violation: This doesn't happen in the standard model.
- C-symmetry and CP-symmetry violation. This DOES happen in the standard model, but the effect is not strong enough. It may happen for neutrinos, as we will study in a future lecture.
- Interactions out of thermal equilibrium.