Neutrino Physics

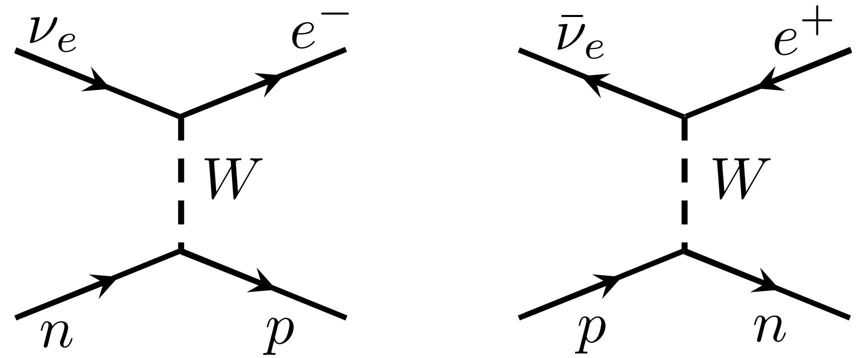
Neutrino physics is at the interface of particle physics and astrophysics. Many experiments designed to observe neutrino properties (e.g. SNO) have made astrophysical observations, and many detectors designed to use neutrinos as astronomical messengers have contributed to the knowledge of particle physics (e.g. IceCube).

- Neutrino-matter interaction
- Neutrino oscillations
- Neutrino mass
- Sterile neutrinos
- CP violation and neutrinos

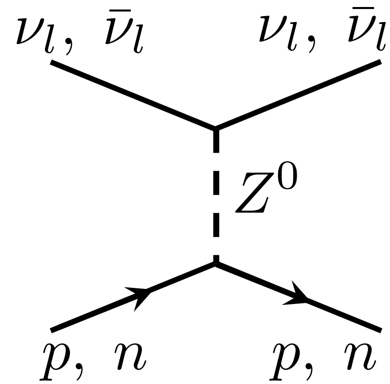
References: PDG
Martin & Shaw, Particle Physics
arXiv:0704.1800 (big review)

Neutrino-matter interaction – low energy (incomplete)

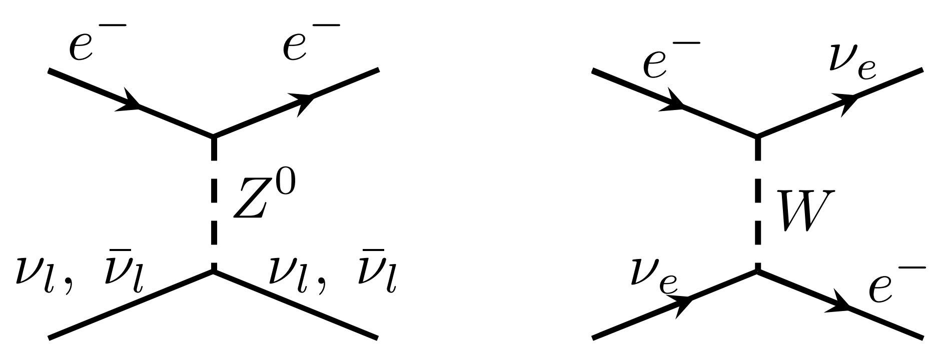
Charged current interactions



Neutral current interactions

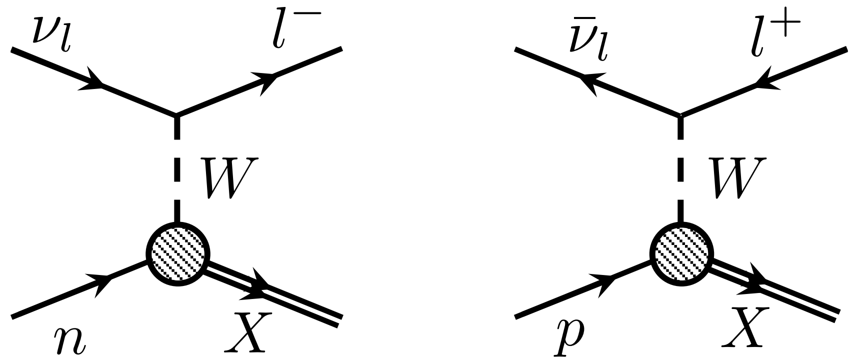


Electron scattering

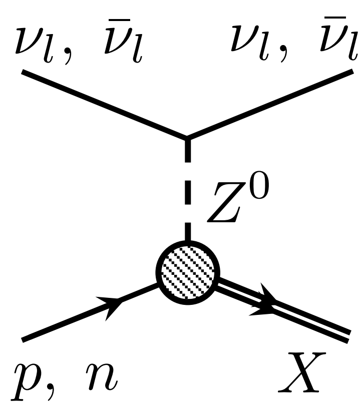


Neutrino-matter interaction – high energy **(incomplete)**

Charged current interactions



Neutral current interactions



For energies high enough, neutrinos probe the structure of nuclei, protons and neutrons.

X : baryons and mesons

Why are weak interactions “weak”?

The fine structure constant is α = 1/137. The weak constant is $\alpha_w \approx 1/240$. (Recall $\alpha \approx e^2$, and each vertex gets a factor of e). It’d would seem that weak interactions should be of similar strength at E.M.

Propagator for photon: $\frac{1}{q^\nu q_\nu}$ -> Coulomb

Propagator for $W^\pm,\ Z^0$: $\frac{1}{q^\nu q_\nu - m^2c^2} \to -\frac{1}{m^2c^2}$

-> Yukawa-like potential

$$V(r) = g\frac{e^{-r/r_0}}{r}$$

(Propagators are missing factors)

$$m_\gamma = 0 \qquad m_W = 80.4\ \ \mathrm{GeV} \qquad m_Z = 91.2\ \ \mathrm{GeV}$$

Effective weak range ~10^{-3} fm (proton size 1 fm).

Zero photon mass gives E.M. infinite range.

Neutrino Oscillations

Flavored neutrinos, $\nu_e,\ \nu_\mu,\ \nu_\tau$, are weak eigenstates. They are NOT the free eigenstates. The free eigenstates are $\nu_1,\ \nu_2,\ \nu_3$.

The generic base 3-dimension base transformation in Quantum Mechanics is of the form (PMNS matrix):

$$U = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

The angles are known as the "mixing angles".
The phase allows for CP violation.

Neutrino Oscillations

Recent values for the mixing angle are:

$$\sin^2\theta_{12} = 0.304 \pm 0.014$$

$$\sin^2\theta_{23} = 0.51 \pm 0.05$$

$$\sin^2\theta_{13} = (2.19 \pm 0.12) \times 10^{-12}$$

Particle Data Group (2016)

Let's imagine a beam of flavor α and fixed momentum. For simplicity, let's assume 2-flavors only. What is the probability of observing flavor β at a distance L from the source?

The initial state is (In 2-D only one angle is needed):

$$|\nu_\alpha >= \cos\theta|\nu_1 > + \sin\theta|\nu_2 >$$

Using the time evolution operator with a free Hamiltonian:

$$e^{-iHt/\hbar}|\nu_\alpha >= \cos_\theta e^{-iE_1 t/\hbar}|\nu_1 > + \sin\theta e^{-iE_2 t/\hbar}|\nu_2 >$$

Neutrino Oscillations

The probability of observing flavor β at a distance L from the source is:

$$P(\nu_\alpha \to \nu_\beta) = |<\nu_\beta|e^{-iHt/\hbar}|\nu_\alpha>|^2$$

The final state is:

$$|\nu_\beta >= -\sin\theta|\nu_1> + \cos\theta|\nu_2>$$

Some trigonometry later:

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2 \frac{(E_2 - E_1)L}{2c\hbar}$$

Note that the neutrino mass is very small, so $E,\ cp >> m_\nu c^2$

Thus:

$$E_2 - E_1 = \sqrt{m_2^2c^4 + c^2|\vec{p}|^2} - \sqrt{m_1^2c^4 + c^2|\vec{p}|^2}$$

$$\sqrt{m^2c^4 + c^2|\vec{p}|^2} \approx cp(1 + \frac{m^2c^2}{2p^2}) \qquad E \approx pc$$

$$E_2 - E_1 \approx \frac{m_2^2 - m_1^2}{2p} \approx \frac{\Delta m_{21}^2}{2E}$$

Neutrino Oscillations

Finally

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2 \frac{\Delta m^2_{21} L}{4c\hbar E}$$

Oscillations are possible <u>only</u> if at least one neutrino has mass.

In general, neutrino oscillations depend on all parameters:

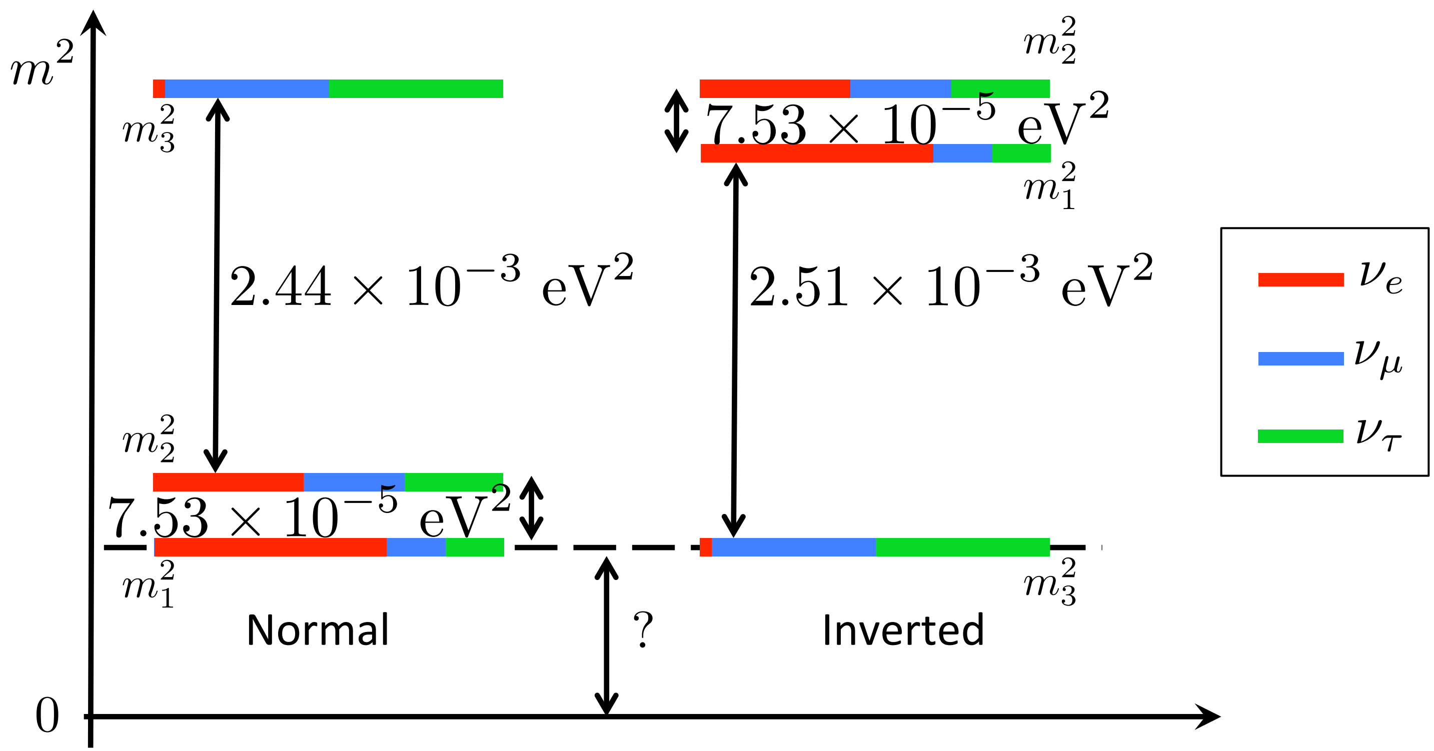
$$\Delta m^2_{21}, \;\; \Delta m^2_{13}, \;\; \theta_{12}, \;\; \theta_{13}, \;\; \theta_{23}, \;\; \delta$$

In many – but not all – experiments, the 2 flavor oscillation case is a reasonable description. Whether 2-flavor oscillations are a good description depends on L/E on the flavors produced by the source and the flavors that want to be detected.

Neutrino Oscillations

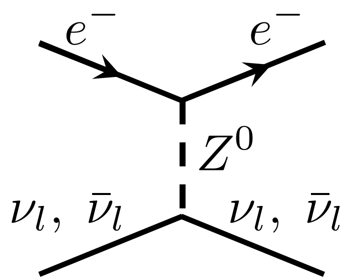
The most recent values for the square difference of masses are shown: (pdg 2016)

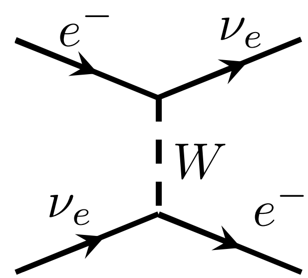
By definition $\Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{13}^2 = 0$, so we don't need to measure the third value.

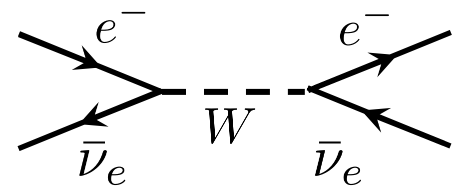


<u>Current</u> experiments can't distinguish among hierarchies.

MSW effect – Neutrino propagation in matter







Neutrinos propagating through matter are not "freely" propagating. They are subject to an effective potential that can be calculated using the diagrams shown above.

The diagram on the left applies to ALL neutrinos and antineutrinos equally. The center diagram applies to ν_e only, and the right diagram applies to $\bar{\nu}_e$ only. The diagrams for ν_e and $\bar{\nu}_e$ are related by time reversal.

The effective potential can be re-interpreted as adding a term to the mass operator.

MSW effect – 2 flavor oscillations

The equation of motion for free neutrinos in the free base is:

($\hbar = 1 \quad c = 1$)

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = H_m \begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

And in this base, the Hamiltonian is:

$$H_m = \frac{1}{2E}\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}$$

Recall: $H_m|\nu_1 >= E_1|\nu_1 >\sim (m_1^2/2p + p)|\nu_1 >$

Adding a potential results in:

$$H = H_m + V$$

V is NOT diagonal in the $|\nu_1>$, $|\nu_2>$ base

MSW effect – 2 flavor oscillations

In particular, in the free base, the potential is:

$$U^\dagger V U = U^\dagger \begin{pmatrix} \pm\sqrt{2} G_F n_e & 0 \\ 0 & 0 \end{pmatrix} U$$

The plus sign is for neutrinos and the negative sign for antineutrinos. G_F is Fermi's weak interaction constant, and n_e is the electron density in the material.

Diagonalizing the new Hamiltonian

$$\frac{1}{2E} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} + U^\dagger \begin{pmatrix} \pm\sqrt{2} G_F n_e & 0 \\ 0 & 0 \end{pmatrix} U$$

Results in:

a) New mass squared terms

b) New "matter" eigenstates, i.e. new mixing angles

MSW effect

MSW is critical for understanding solar neutrino oscillations.

MSW is also critical in understanding MeV supernovae neutrinos.

The MSW effect may help measure the hierarchy. This is because the effect is different for ν_e and $\bar{\nu}_e$. Because the diagrams are related by time-reversal, the effective potential changes sign when going from ν_e to $\bar{\nu}_e$

Vacuum oscillations for astrophysical objects

It is common for high-energy neutrinos to be produced from pion decay:

$$\pi^+ \to \mu^+ + \nu_\mu \to e^+ + \nu_e + \nu_\mu + \bar{\nu}_\mu$$

$$\pi^- \to \mu^- + \bar{\nu}_\mu \to e^- + \bar{\nu}_e + \nu_\mu + \bar{\nu}_\mu$$

And the kinematics are such that all three neutrinos have approximately the same energy. Thus the flavor flux ratios, at the astrophysical source are:

$$\Phi_{\nu_e+\bar{\nu}_e} : \Phi_{\nu_\mu+\bar{\nu}_\mu} : \Phi_{\nu_\tau+\bar{\nu}_\tau} = 1 \ : \ 2 \ : \ 0$$

In the limit of very long baseline (i.e. astrophysical sources)

$$\sin^2 \frac{L\Delta m^2_{ij}}{4E\hbar} \to 1/2$$

J Learned, S Pakvasa
Astropart Phys 3 (1995) 267

So at Earth we see (easy matrix-vector multiplication):

$$\Phi_{\nu_e+\bar{\nu}_e} : \Phi_{\nu_\mu+\bar{\nu}_\mu} : \Phi_{\nu_\tau+\bar{\nu}_\tau} = 1 \ : \ 1 \ : \ 1$$

There are exceptions, e.g. T Kashti, E Waxman PRL 95 (2005) 181101

Vacuum oscillations for astrophysical neutrinos

For astrophysical neutrinos (unproven here):

$$P(\nu_\alpha \to \nu_\beta) = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

So for astrophysical baselines, oscillations are independent of Δm^2. With the current knowledge of mixing angles:

	ν_e	ν_μ	ν_τ
ν_e	60%	20%	20%
ν_μ	20%	40%	40%
ν_τ	20%	40%	40%

Example: Imagine that neutrinos are produced via neutron decay in an astrophysical source (this is true for low energy GZK neutrinos), the Earth flavor flux ratio is 0.6:0.2:0.2

Neutrino mass – tritium decay

Neutrino mass can in principle be measured from the end point of β-decay spectrum. This is typically done with tritium

$$^3H \rightarrow ^3He + e^- + \bar{\nu}_e$$

From our special relativity lecture:

$$E_e^{\max} = \frac{M_{^3H}^2 + m_e^2 - (M_{^3He} + m_{\bar{\nu}})^2}{2M_{^3H}} c^2$$

Current limit: 2 eV (Particle Data Group 2016)

Why tritium: low half life (12.3 yr), low end point energy (18.57 keV), atomic states calculable exactly, etc.

Neutrino mass can also be constrained cosmologically or astro-physically.

Neutrino Mass

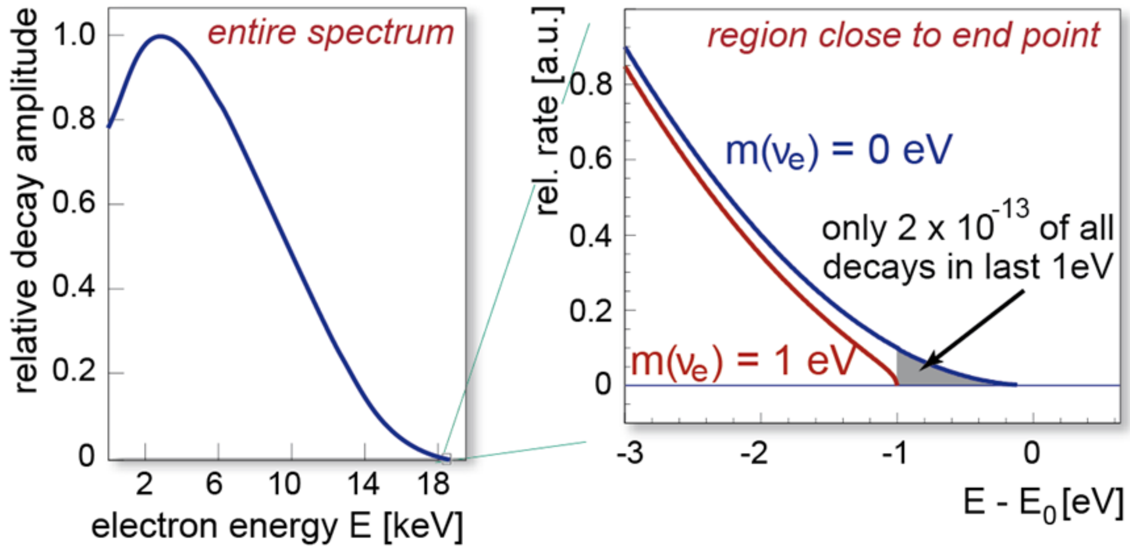


Figure: Lutz Bornschein
Recontrois de Blois Conf 2012

Katrin can measure down to 0.35 eV and limit to below 0.2 eV

Wait, doesn't tritium decay measure $m_{\bar{\nu}_e}$?

Strictly speaking $\bar{\nu}_e$ doesn't have a mass. So what is that is being measured in tritium decay?

What is being measured is $m_{\bar{\nu}_e} =< \bar{\nu}_e|M|\bar{\nu}_e >$

Because $\begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix} = U \begin{pmatrix} \bar{\nu}_1 \\ \bar{\nu}_2 \\ \bar{\nu}_2 \end{pmatrix}$ $\quad U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$

Then the ket is:

$$|\bar{\nu}_e >= \sum_i U_{ei}|\bar{\nu}_i >$$

So

$$< m_{\bar{\nu}_e} >= \sum_i \sum_j < \bar{\nu}_i|U_{ei}^* M U_{ei}|\bar{\nu}_j >$$

Wait, doesn't tritium decay measure $m_{\bar{\nu}_e}$?

Since the mass operator is diagonal in the free state base:

$$< m_{\bar{\nu}_e} > = \sum_i \sum_j U^*_{ei} U_{ej} m_j < m_{\bar{\nu}_i} | m_{\bar{\nu}_j} >$$

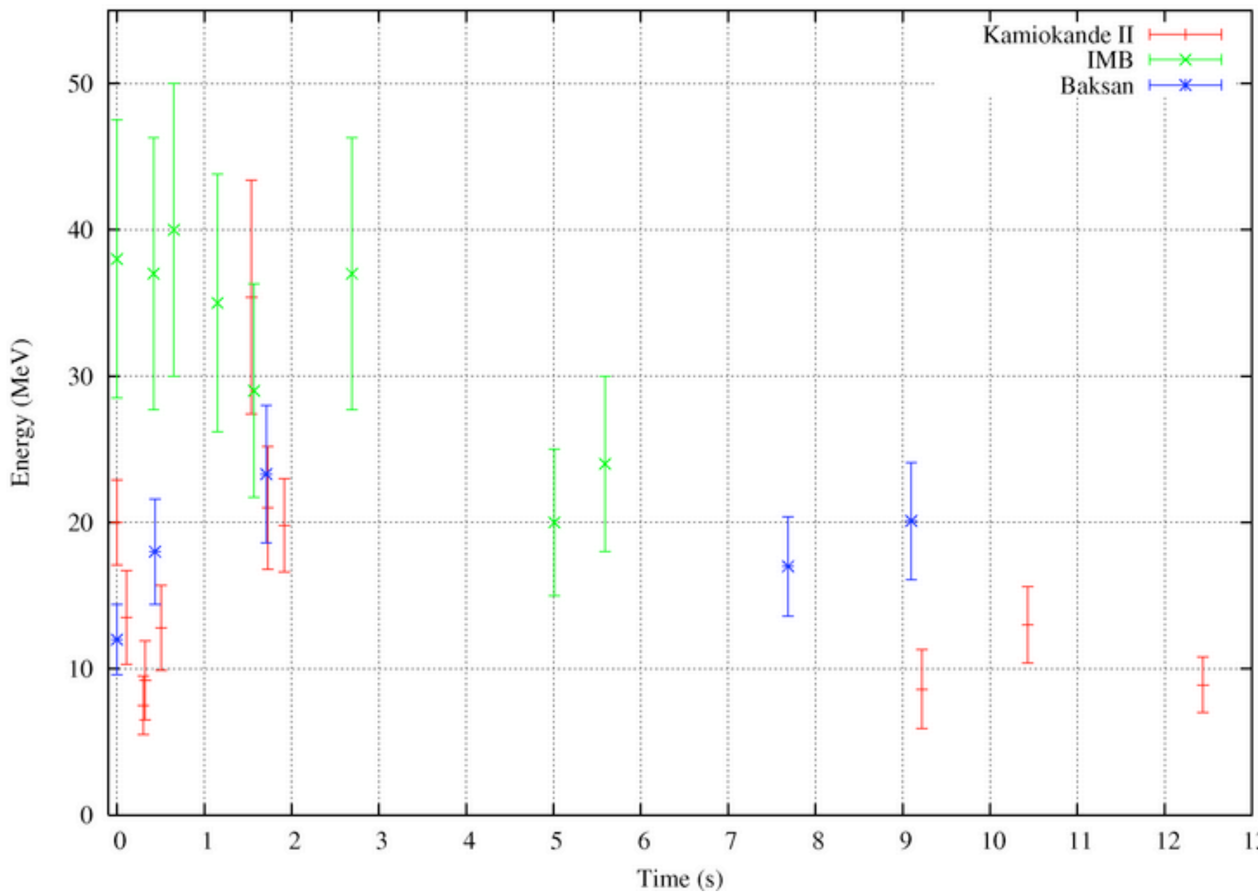
$$= \sum_i \sum_j U^*_{ei} U_{ej} m_j \delta_{ij}$$

Then experiments that look at the end point of beta decay, measure:

$$< m_{\bar{\nu}_e} > \sum_i |U_{ei}|^2 m_i$$

Neutrino mass limit from SN 1987A

In February 1987 a core collapse supernova was observed in the LMC. A total of ~25 neutrinos were reported by 3 detectors. These are thought to be mostly $\bar{\nu}_e$.



Neutrino mass limit from SN 1987A

The LMC is at 52 kpc. At the speed of light, the travel time is $t_0 = 5.3 \times 10^{12}$ s.

If neutrinos have mass m, then you expect the total travel time to be $t_{\rm obs} - t_{\rm em} = t_0(1 + m^2/2E^2)$. Assuming neutrinos are all produced simultaneously, there's an energy dependent spread in arrival times.

TABLE I. Observation times and inferred neutrino energies in the Kamioka experiment.

Event No.	$t_{\rm obs}$ (s)	Energy (MeV)
1	0 (def)	21.3 ± 2.9
2	0.107	14.8 ± 3.2
3	0.303	8.9 ± 2.0
4	0.324	10.6 ± 2.7
5	0.507	14.4 ± 2.9
6[a]	0.686[a]	7.6 ± 1.7[a]
7	1.541	36.9 ± 8.0
8	1.728	22.4 ± 4.2
9	1.915	21.2 ± 3.2
10	9.219	10.0 ± 2.7
11	10.433	14.4 ± 2.6
12	12.439	10.3 ± 1.9

[a]Event 6 was excluded from the analysis of Ref. 3 because it had fewer than 20 photomultiplier hits.

W.D. Arnett & J.L Rosner
Phys. Rev. Lett. 58, 1906–1909 (1987)

Since this was not seen:

$$m_{\bar{\nu}_e} < 12 \text{ eV}$$

Atmospheric Neutrinos

Air showers product of cosmic ray interactions with nitrogen and oxygen in the upper (10-20 km) atmosphere result in atmospheric neutrinos. Neutrinos are product of the decay of pions and heavier mesons, such as kaons

$$\pi^+ \to \mu^+ + \nu_\mu; \quad \mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$ (And similar for π^-)

The contribution from kaons increases with neutrino energy.

The decay length of a 1 GeV $\mu^\pm$ is 6 km – less than the atmosphere height. So, $\mu^\pm$ and mesons decay before they reach the ground. It follows that the flavor flux ratio is 1:2:0.

$\pi^\pm$ interaction length ($\lambda_{\pi^\pm} \approx 62 \text{ g} \cdot \text{cm}^{-2}$) is shorter than the decay length ($\gamma c \tau$) for ε_π = 115 GeV. So $K^\pm$ are the major source of $\nu_\mu \;\&\; \bar{\nu}_\mu$ at higher energy. For $K^\pm$, interaction is more important than decay above ε_K = 850 GeV.

Atmospheric Neutrinos

The energy spectrum of the primary cosmic rays is $E^{-\alpha}$. The interaction of mesons in the atmosphere steepens this spectrum to $E^{-(\alpha+1)}$.

At low energies, muons are the main source of $\nu_e + \bar{\nu}_e$. At high energies it's $K_L^0 \to \pi^{\pm} + e^{\mp} + \bar{\nu}_e(\nu_e)$. Once K_L^0 is dominant, the flavor flux ratio of $\nu_e + \bar{\nu}_e$ decreases significantly. At ~TeV energies, the flavor flux ratio is ≈ 1:10:0

At even higher energies (100 TeV – 1 PeV), the dominant source of atmospheric neutrinos are charmed mesons (e.g. $D^{\pm}$ mesons). These mesons have very short lifetime, hence these neutrinos are called prompt. Because charmed mesons do not interact before decaying, the neutrino spectrum goes as $E^{-\alpha}$.

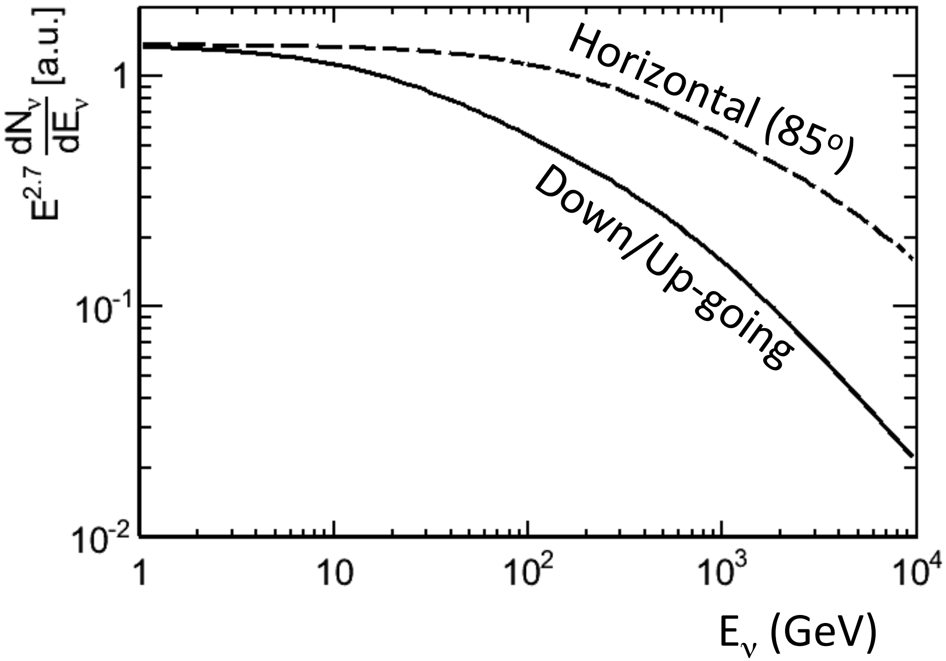
Atmospheric Neutrinos

The muon neutrino flux can be parameterized (Gaisser '90) as:

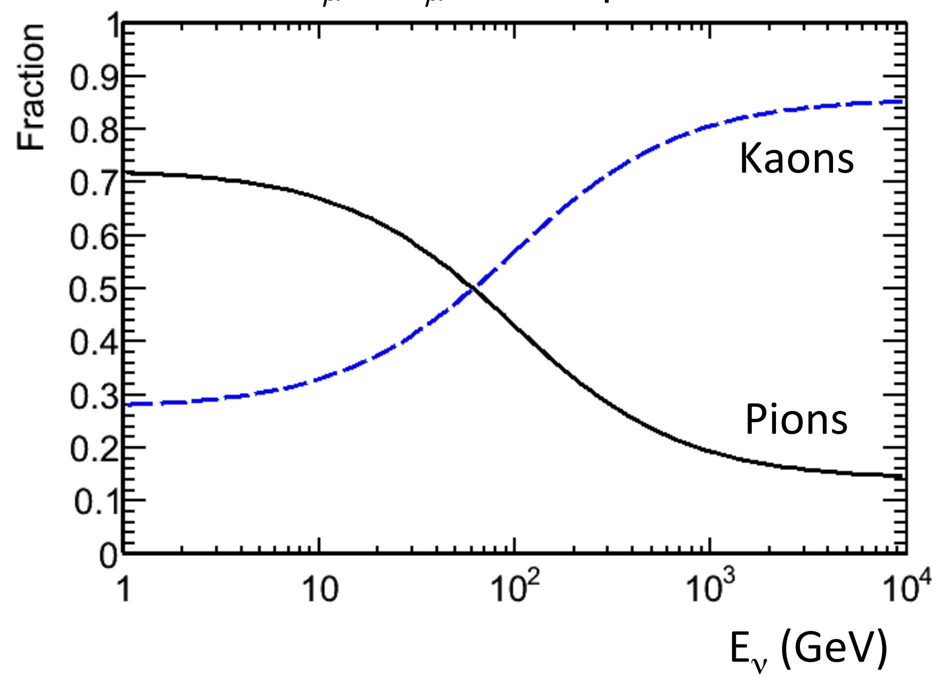
$$\frac{dN_\nu}{dE_\nu} \approx 0.0096 E_\nu^{-2.7} \left[\frac{1}{1 + \frac{3.7 E_\nu \cos\theta}{115 GeV}} + \frac{0.38}{1 + \frac{1.7 E_\nu \cos\theta}{850 GeV}} \right]$$

Detailed calculations by, e.g. Honda et al. Phys. Rev. D 75, 043006 (2007)

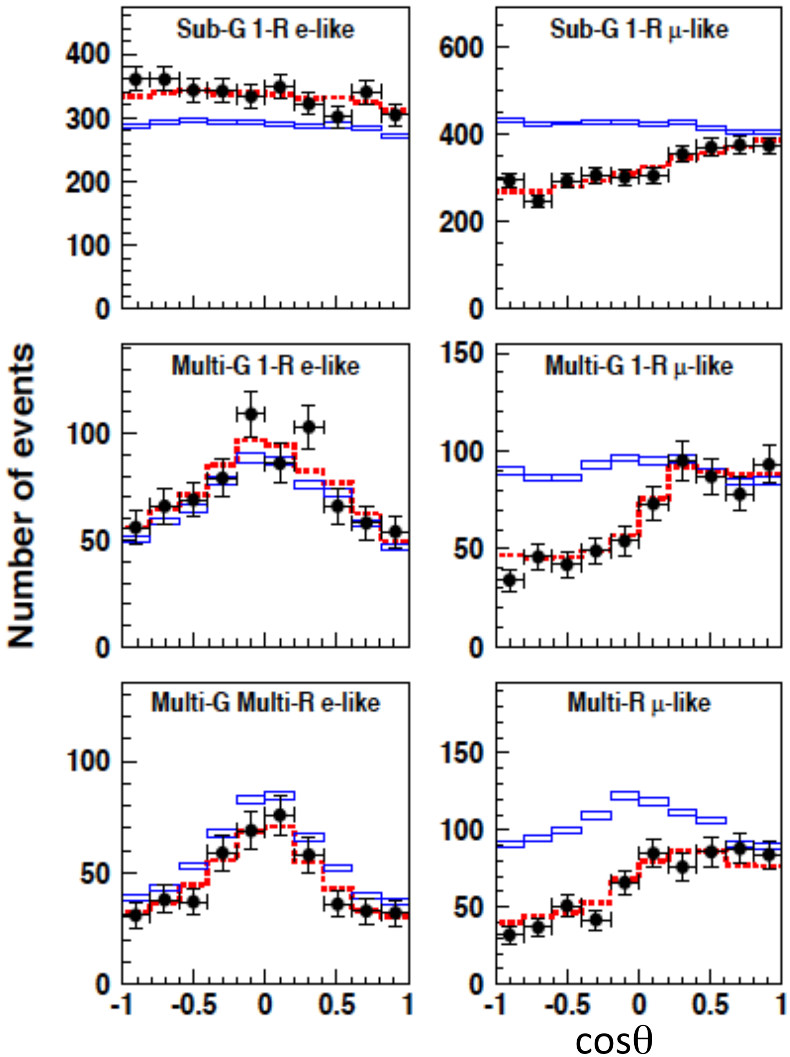
Flux of $\nu_\mu + \bar{\nu}_\mu$



Fraction of $\nu_\mu + \bar{\nu}_\mu$ due to pions and kaons



Super-K atmospheric neutrino oscillations.



Black: data

Red: fit to data with oscillations

Blue: No oscillations
Sub-G: < 1 GeV ; Multi-G: > 1 GeV
e-like: ν_e ; μ-like: ν_μ
1-R: one Cherenkov ring
Multi-R: more than one Cherenkov ring

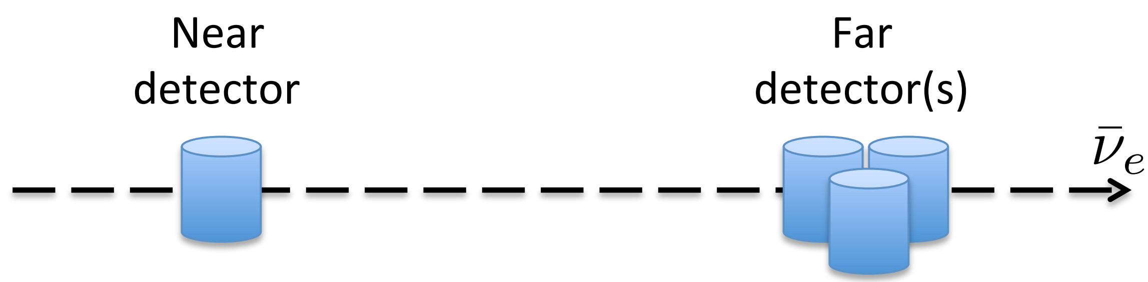
Super-K Phys.Rev.D74:032002,2006

Atmospheric neutrinos (in the GeV scale) are mostly sensitive to θ_{23} and Δm^2_{23}

Daya Bay and Neutrino Oscillations

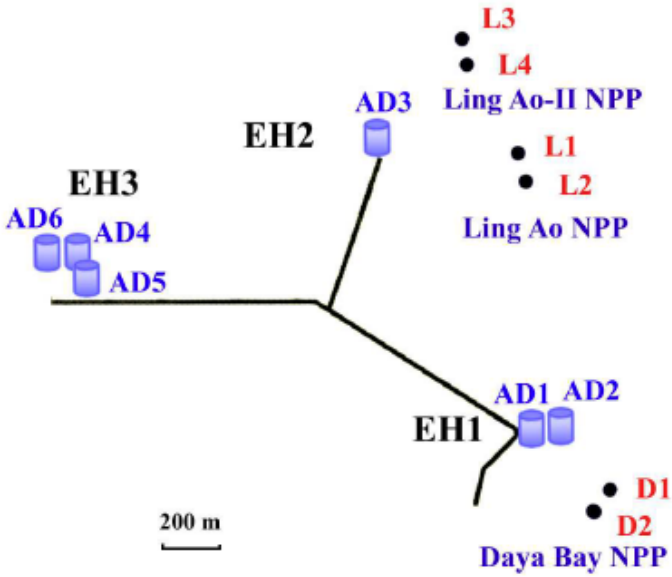
Short baseline neutrino detectors (~1 km) with nuclear reactor neutrinos (a few MeV) are mostly sensitive to θ_{13}.

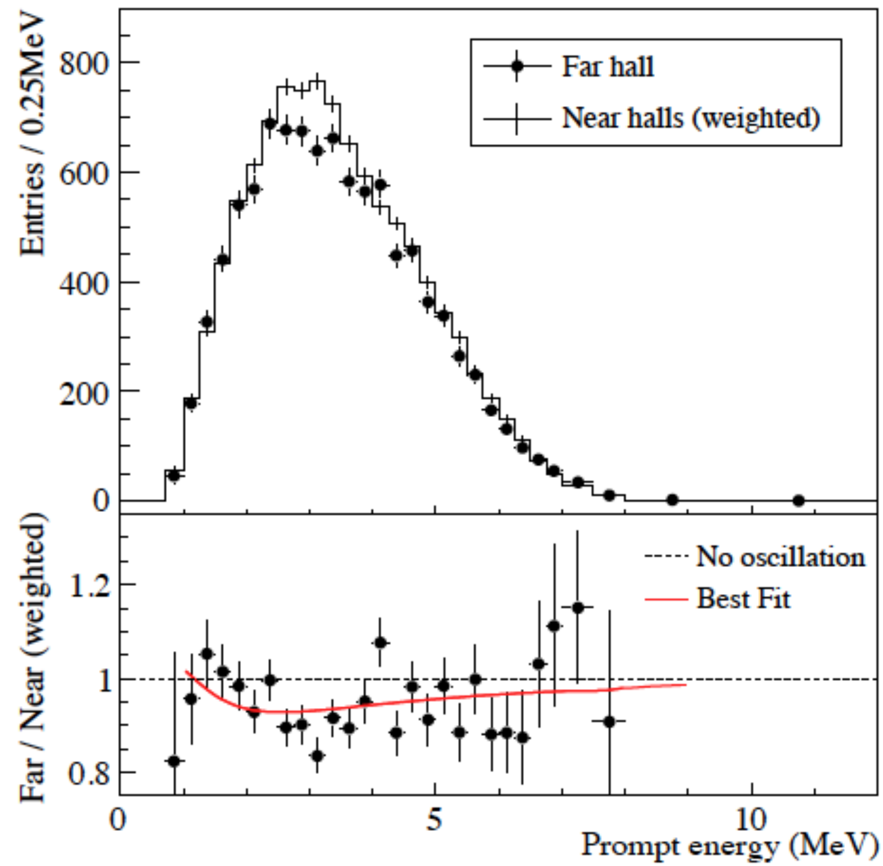
The first non-zero measurement of this mixing angle was done by Daya Bay and has been confirmed by several other experiments.



Oscillations are observed as a disappearance of $\bar{\nu}_e$ in the far detector. Systematics are controlled by measuring the ratio of neutrinos in the far to the near detector.

Daya Bay and Neutrino Oscillations





Daya Bay. arXiv:1203.1669

This result is particularly interesting because it enables CP violation with neutrinos.

Sterile neutrinos

LEP showed that there are only 3 flavors of neutrinos that couple to Z^0. Any additional neutrino must be either sterile (only gravitational interaction) or have a mass $m_\nu > m_{Z_0}$

A short baseline, target experiment, LSND, reported the observation of excess $\bar{\nu}_e$ in a beam of $\bar{\nu}_\mu$. On it's own this is best interpreted as oscillations $\bar{\nu}_\mu \to \bar{\nu}_e$ with $\Delta m^2 \sim 1\ \mathrm{eV}$.

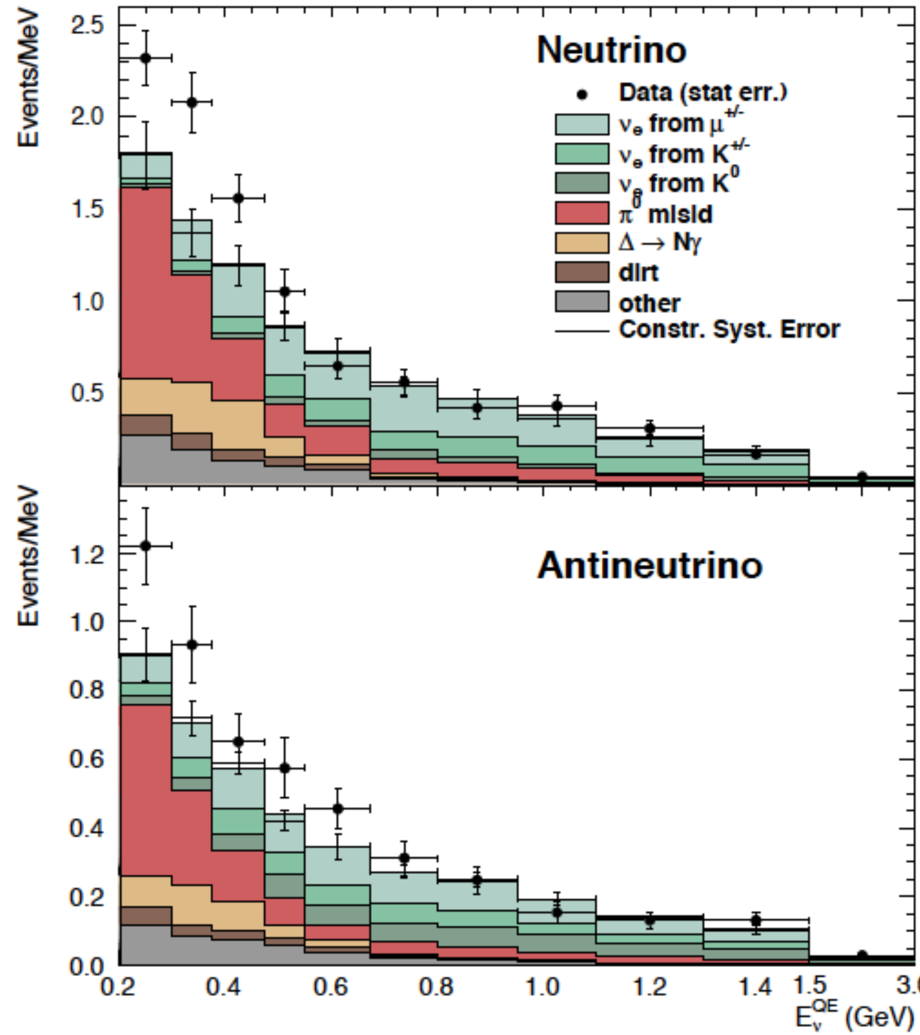
An sterile neutrino is required if LSND results are included into all other oscillation results.

LSND has been repeated for ν_μ and $\bar{\nu}_\mu$ beams by the miniBoone collaboration.

miniBoone results

miniBoone (and LSND) use a muon neutrino beam from pion decay produced in target experiments at accelerators. A residual flux of electron neutrinos is expected.

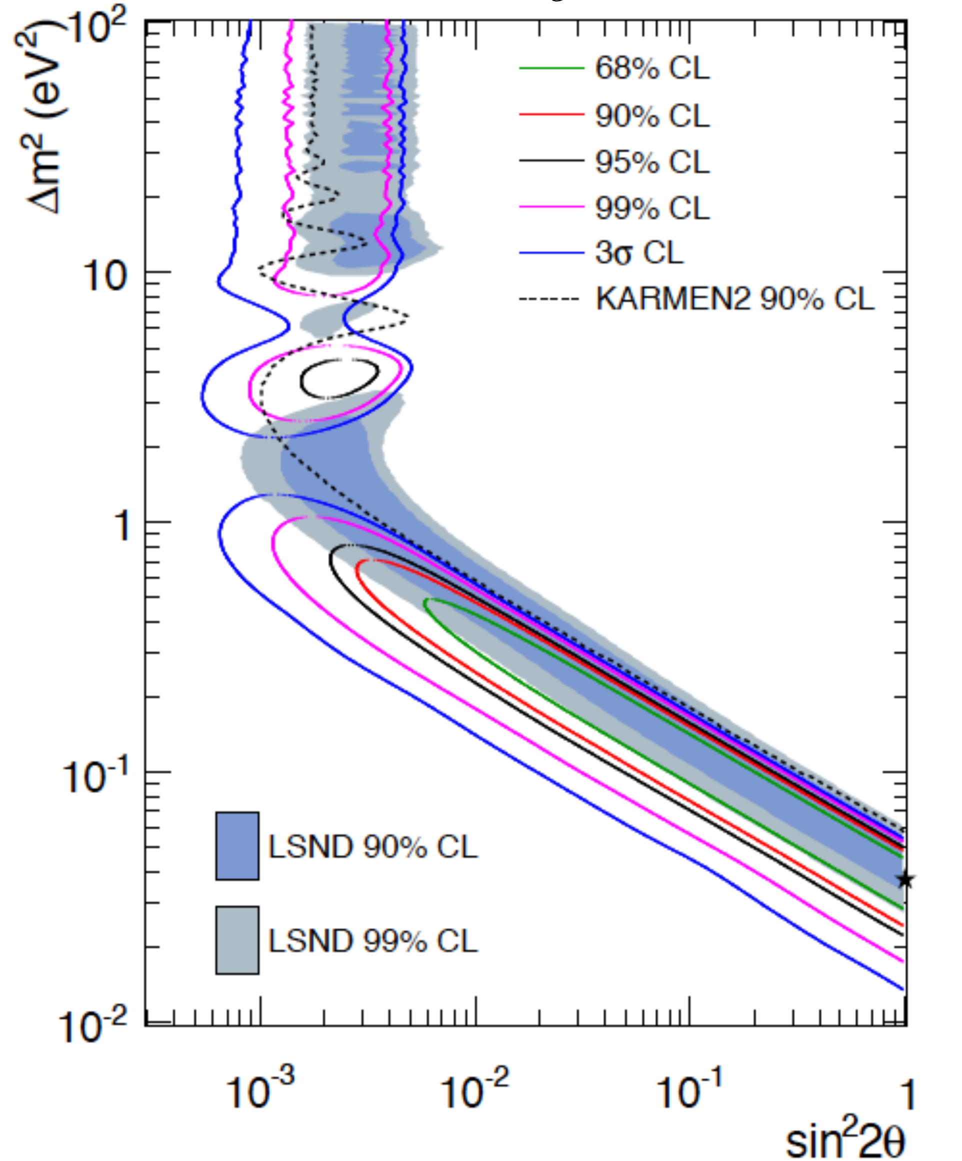
This result is derived from an excess of 3.8 σ of electron neutrinos and antineutrinos.



miniBoone arXiv:1207.4809

miniBoone results

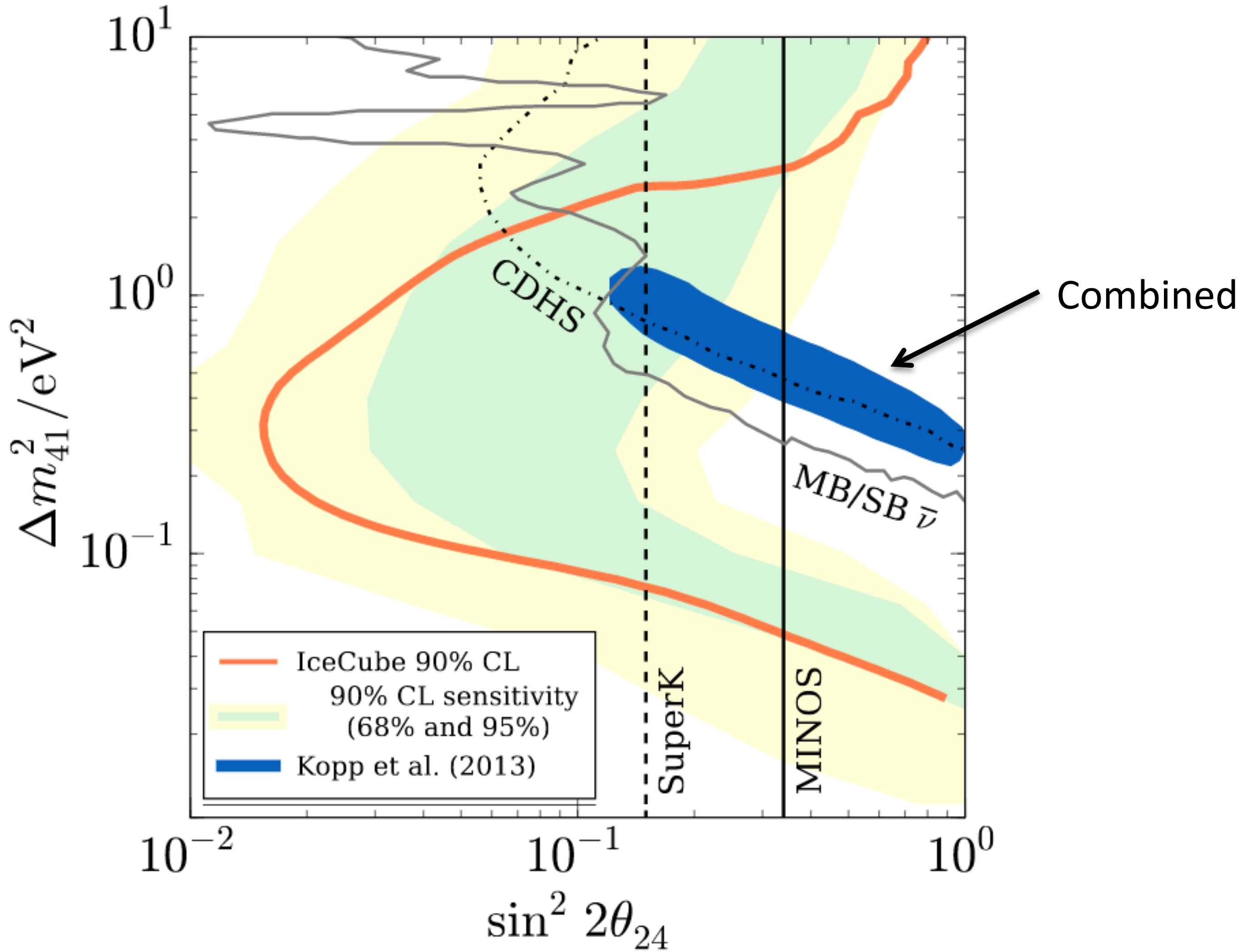
Combined ν_e and $\bar{\nu}_e$ allowed regions.



These results use the full neutrino energy range of 200 MeV to 3 GeV.

miniBoone arXiv:1207.4809

But IceCube doesn't see anything



PRL 117 (2016) 071801