The Expanding Universe

Distance Ladder & Hubble's Law

Robertson-Walker metric

Friedman equations

Einstein – De Sitter solutions

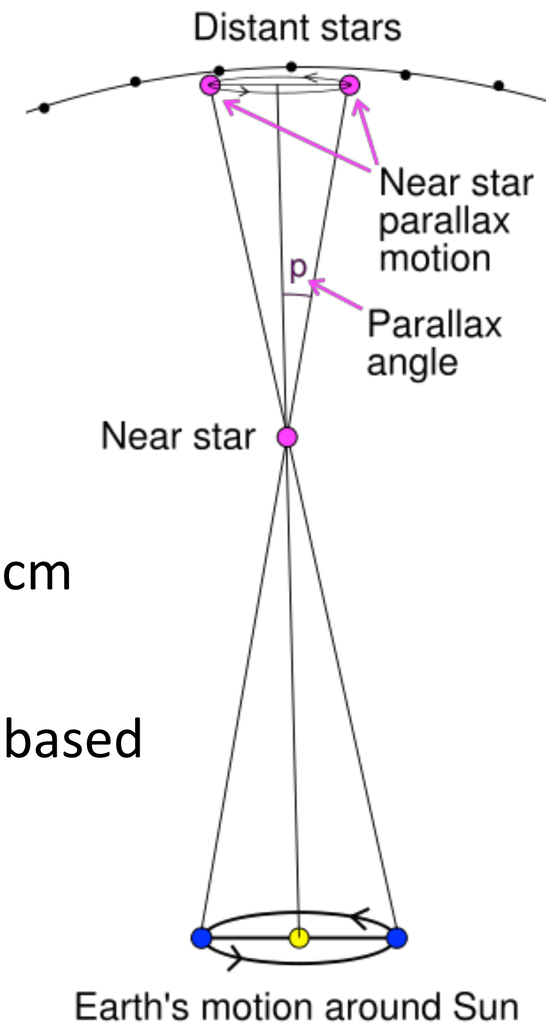
Cosmological distance

Observed properties of the Universe

The distance ladder

Measuring distances in astronomy is hard. The most direct way of measuring distance is parallax.

$$d = \frac{1\ \mathrm{AU}}{\tan p} \approx \frac{1}{p}\ \mathrm{AU}$$



The parsec is the distance at which a parallax of 1” is observed.

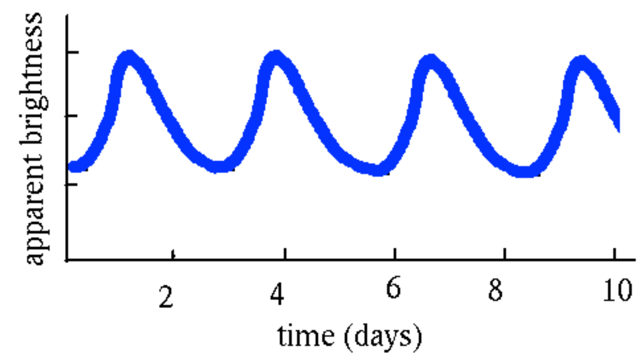
1 pc = 2.06×10^{5} AU = 3.26 light-year = 3.09×10^{18} cm

State of the art in measuring parallaxes is space based (Gaia satellite – successor to Hipparchus).
Up to 10 kpc distances can be measured with 10% precision

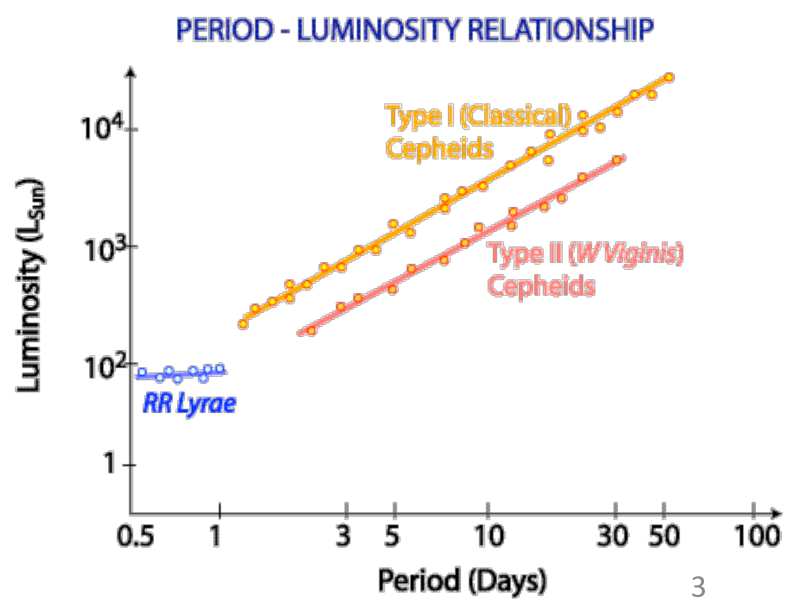
Cepheid variable stars

Cepheid stars are variable stars. They are not main sequence stars. Instead they are red giants that undergo oscillations in size, and hence in luminosity. The period of the oscillations varies from a few days to a few months.

Stefan-Boltzman $L = \sigma A T^4$



Henrietta Leavitt noticed a relationship between average luminosity and period this was done with Cepheid stars in the SMC and LMC.



Cepheids were used by Edwin Hubble to measure the distance to nearby galaxies.

Hubble's law and redshift

Hubble observed that the redshift (receding speed) of galaxies was linearly correlated with distance.

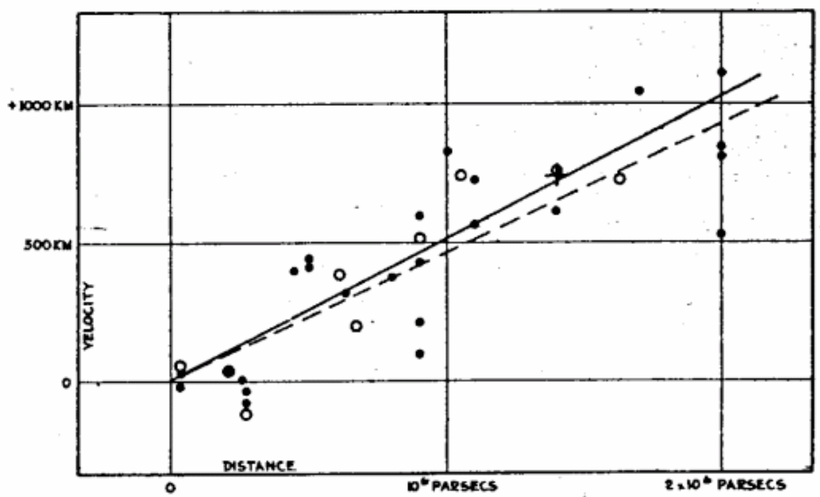


FIGURE 1

Redshift is now a distance indicator

$$\lambda' = \lambda(1+z)$$

$$\lambda' = \lambda\sqrt{\frac{1+\beta}{1-\beta}}$$

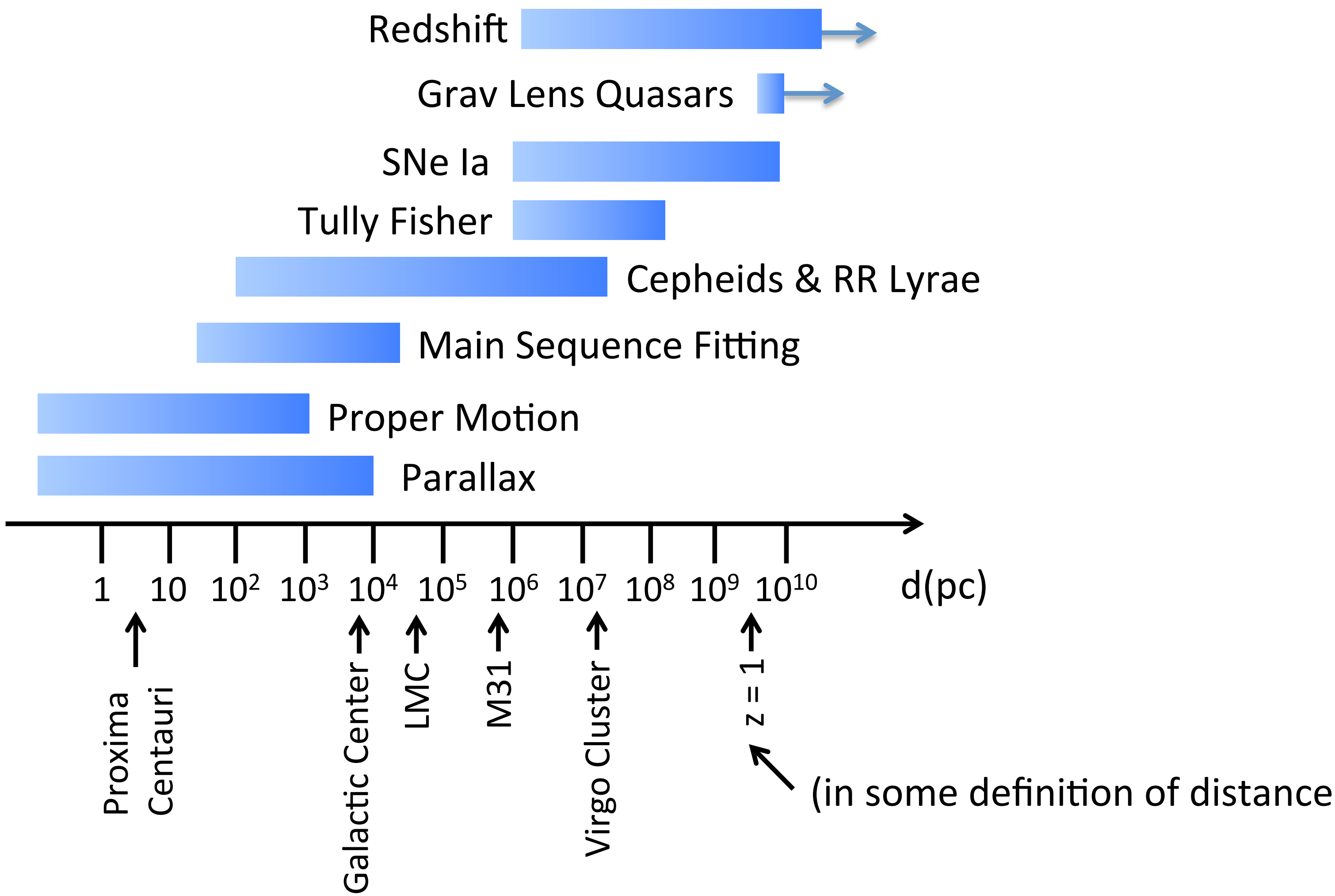
$$\lambda' \rightarrow \lambda(1+\beta) \qquad \text{(nearby)}$$

The Universe is expanding.

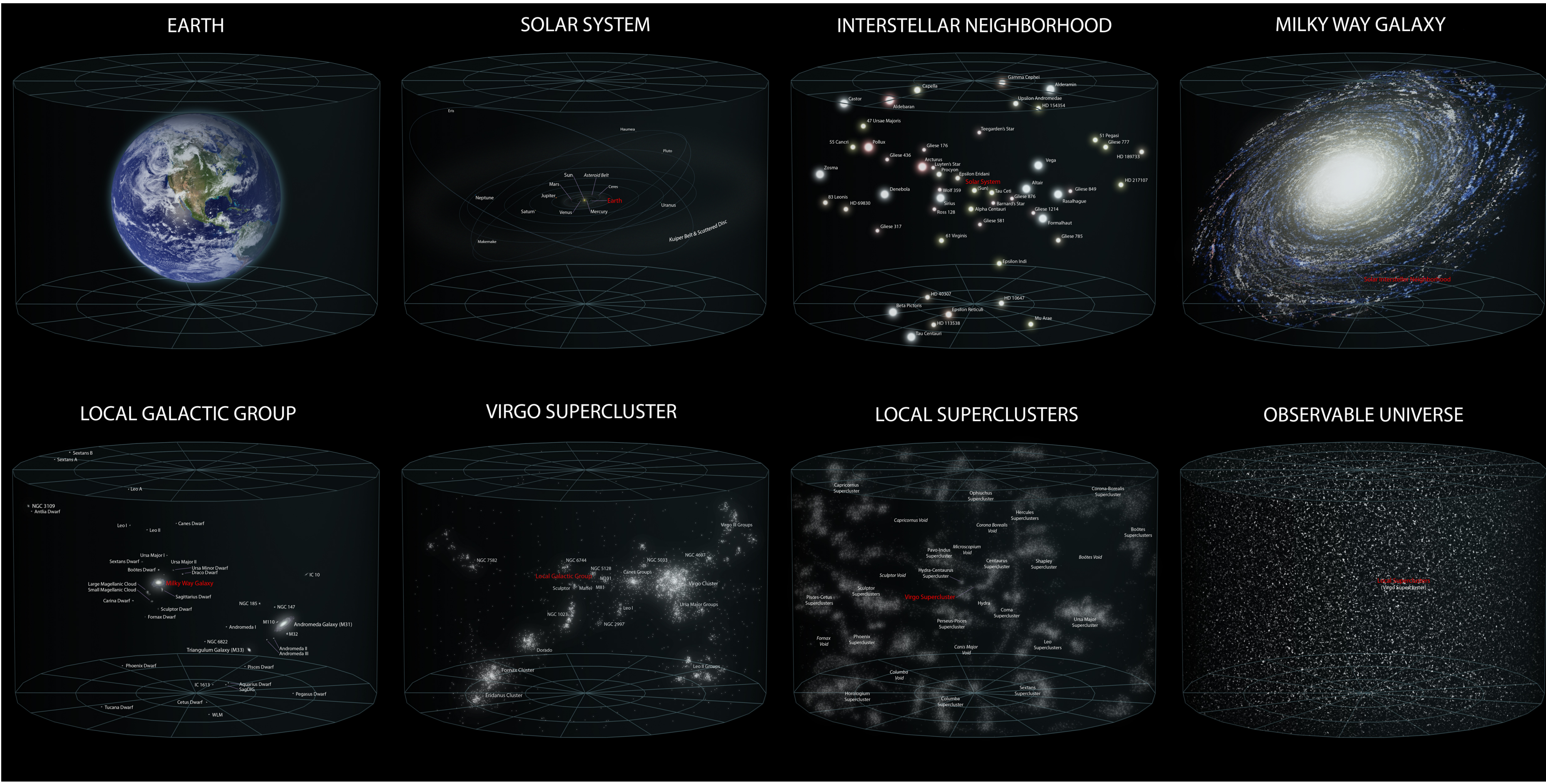
This relationship is Hubble's law: $v = H_0 D$.
With Cepheids, and using the Hubble space telescope:
H_0, Hubble's constant, is 73.00 ± 1.75 km.s^{-1}.Mpc. (arXiv:1604.01424)
Because the Hubble constant has (traditionally) been uncertain, it is common to see the definition $H_0 = hH$, with H = 100 km.s^{-1}.Mpc.

More on the distance ladder

There are many more steps in the distance ladder …



Distances


EARTH
SOLAR SYSTEM
INTERSTELLAR NEIGHBORHOOD
MILKY WAY GALAXY
LOCAL GALACTIC GROUP
VIRGO SUPERCLUSTER
LOCAL SUPERCLUSTERS
OBSERVABLE UNIVERSE

From Special to General relativity

Special relativity – Inertial frames:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 dt^2 - dr^2 - r^2(d\theta + \sin^2\theta d\phi^2)$$

Take an inertial frame S and an accelerated frame S'.
S' has <u>small</u> acceleration $a\hat{\imath}$ with respect to S.

$$S:\ x, y, z, t \qquad S':\ x', y', z', t'$$

Then $x' = x - 1/2at^2$

Now

$$dx = \frac{\partial x}{\partial x'}dx' + \frac{\partial x}{\partial t}dt = dx' + at\ dt$$

So

$$ds^2 = (c^2 - a^2t^2)dt^2 - 2atdx'dt - dx'^2 - dy'^2 - dz'^2$$

For a clock in S': $ds^2 = c^2 d\tau^2 = c^2 dt'^2 = (c^2 - 2ad)dt^2$

Now all the way General relativity: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$

The Robertson-Walker metric

General Relativity provides a framework to describe the Universe expansion. The Robertson-Walker metric is

$$ds^2 = (cdt)^2 - a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right]$$

Here a(t) is dimensionless and known as the expansion parameter. The constant K is the Gaussian curvature and has units 1/length^2. K=0 is for an Euclidean universe. K>0 positive curvature, K<0 negative curvature.

The above metric uses <u>comoving</u> coordinates, which are the coordinates on which the *Cosmological Principle* applies. Cosmological Principle: the Universe is homogeneous and isotropic, at least to first approximation.

Comoving time is the time since the Big Bang as measured in a clock that follows the Hubble flow.

Friedman Cosmological Equations

Using Einstein's equation and assuming the Universe is filled with a homogeneous and isotropic fluid of density ρc^2 and pressure p, results in:

$$\frac{\ddot{a}}{a} = \frac{\Lambda}{3} - \frac{4\pi G}{3}(\rho + 3\frac{p}{c})$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8}{3}\pi G\rho - \frac{K}{a^2} + \frac{\Lambda}{3}$$

In the equations above, both p and ρ are functions of the comoving time too. In general, an equation of state relating pressure and density is also needed. Adiabatic expansion of the Universe is assumed so

$$d(\rho c^2 a^3)/da^3 = -p$$

Note that the expansion of the Universe is measured by the Hubble parameter (not a constant anymore)

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

Critical Density

In the case of a flat Universe (K=0) and no cosmological constant

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G\rho}{3} = 0$$

The density takes the value:

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

This is known as the critical density. It's value is:

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} = 1.88 \times 10^{-29} h^2 \text{ g} \cdot \text{cm}^{-3}$$

The present estimate of the baryonic density in the Universe is:

$$\rho_{B,0} = 3 \times 10^{-31} \text{ g} \cdot \text{cm}^{-3}$$

Density parameter – ΛCDM cosmology

The ratio of the density to the critical density is the density parameter

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)}$$

The current baryonic matter density parameter is

$$\Omega_B h^2 = 0.02205$$

There is a (present) contribution from dark matter:

$$\Omega_{cdm} h^2 = 0.1199$$

And a (present) contribution from dark energy:

$$\Omega_\Lambda = 0.6866$$

With h = 0.673 – the Universe is flat.
(Note that h had a different value in a previous slide)

Planck Cosmological parameters: arXiv 1502.01589

Einstein – De Sitter Model

Equation $\dot{a}^2 + Kc^2 = \frac{8\pi G}{3}\rho a^2$ (no cosmological constant) can be re-written thus:

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3}\rho\left(\frac{a}{a_0}\right)^2 = -\frac{Kc^2}{a_0^2}$$

A generic equation of state takes the form $p = \omega\rho c^2$

ω=0 represents pressure-less material (dust) this is also a good representation of a non-relativistic ideal gas.

ω=1/3 represents a radiative fluid (e.g. photons)

ω=-1 represents dark energy – the cosmological constant

Einstein – De Sitter Model

From $d(\rho c^2 a^3) = -p da^3$ and $p = \omega \rho c^2$ we get

$$\rho a^{3(1+\omega)} = \rho_0 a_0^{3(1+\omega)}$$

We can use this in equation (no cosmological constant)

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3}\rho\left(\frac{a}{a_0}\right)^2 = -\frac{Kc^2}{a_0^2}$$

So that

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{\rho}{\rho_c}H_0^2 = \left(\frac{\dot{a}}{a}\right)^2 - \frac{\rho_0}{\rho_c}H_0^2\left(\frac{a_0}{a}\right)^{3(1+\omega)} = -\frac{Kc^2}{a_0^2}$$

In the case of K=0, which implies $\Omega_0 = \rho_0/\rho_c = 1$

$$\left(\frac{\dot{a}}{a}\right)^2 - H_0^2\left(\frac{a_0}{a}\right)^{3(1+\omega)} = 0$$

Einstein De Sitter Model

The K=0 solution is:

$$a(t) = a_0 \left(\frac{t}{t_0}\right)^{2/3(1+\omega)}$$

For Ω < 1 (open Universe, K=-1) and ω=0 the parametric solution is:

$$a(x) = a_0 \frac{\Omega}{2(1-\Omega)} (\cosh x - 1)$$

$$t(x) = \frac{1}{2H_0} \frac{\Omega}{(1-\Omega)^{3/2}} (\sinh x - x)$$

And for Ω>1 (closed Universe, K=1) and ω=0 the parametric solution is:

$$a(x) = a_0 \frac{\Omega}{2(1-\Omega)} (1 - \cos x)$$

$$t(x) = \frac{1}{2H_0} \frac{\Omega}{(1-\Omega)^{3/2}} (x - \sin x)$$

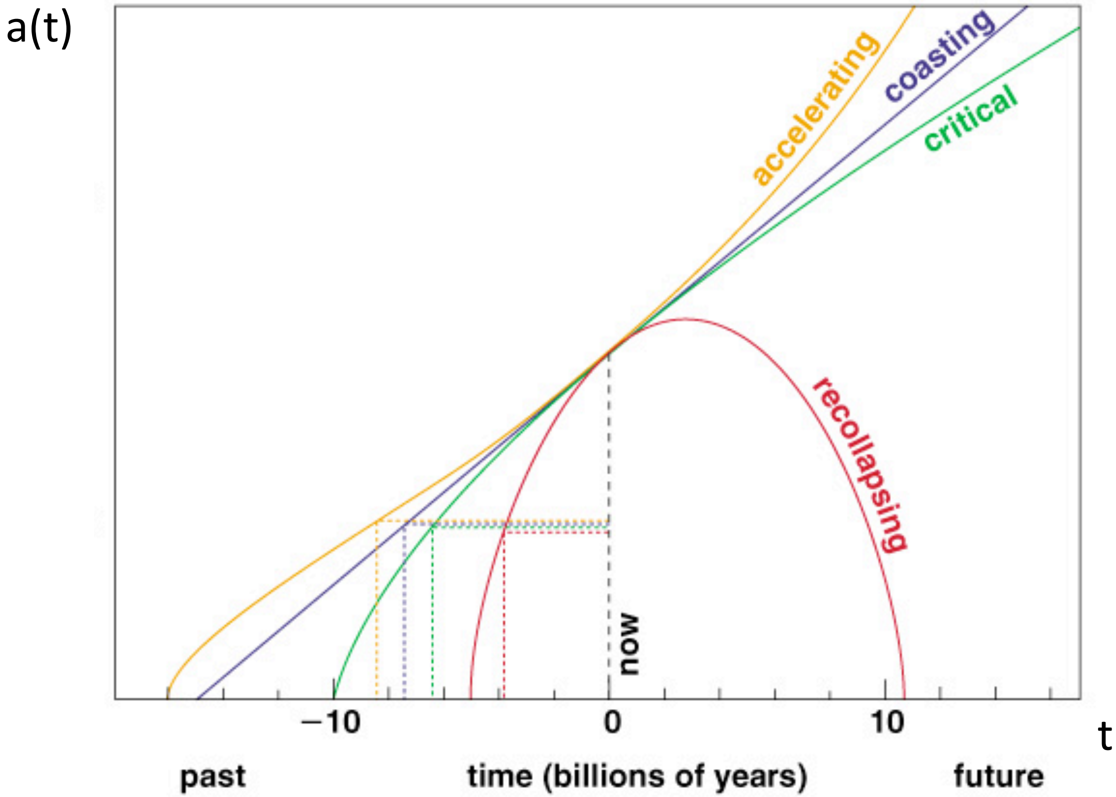
Dark Matter dominated Universe

For K=0 and ω=-1

$$a(t) \propto e^{\sqrt{\Lambda/3t}}$$

ω=1 is not the only possibility and it doesn't even have to be constant with proper time t. The best measurement of w, assuming that it is constant is $\omega = 1.00 \pm 0.06$.

Einstein – De Sitter Universe



The ω=1/3 general solution is:

$$a(t) = a_0(2H_0\Omega_0^{1/2}t)^{1/2}\left(1+\frac{1-\Omega_0}{2\Omega_0^{1/2}H_0t}\right)^{1/2}$$

Luminosity and proper distance

Let's return to the Robertson-Walker metric

$$ds^2 = (cdt)^2 - a(t)^2 \left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]$$

The proper distance assume that the two ends are measured simultaneously in proper time. If at t_0 two events have a proper distance r_0, then at time t they have a proper distance $a(t)/a(t_0)\ r_0$. But proper distance is not directly measurable in practice.
Let's look at a different definition of distance.

The redshift of a photon is defined as:

$$z = \frac{\lambda_o - \lambda_e}{\lambda_e}$$

Where λ_e is the emitted wavelength and λ_0 is the observed wavelength.

Luminosity distance

Photons travel along <u>null geodesics</u>, $ds^2 = 0$. This implies:

$$\int_{t_e}^{t_0} \frac{cdt}{a(t)} = \int_R^0 \frac{dr}{\sqrt{1 - Kr^2}} = f(r)$$

Light emitted the source at $t'_e = t_e + \Delta t_e$ is observed at $t'_0 = t_0 + \Delta t_0$. Assuming small Δt_e and Δt_0, f(r) does not change. Here Δt_e can represent, e.g. the frequency of emitted light.

Then,

$$\int_{t_e}^{t_0} \frac{cdt}{a} = \int_{t'_e}^{t'_0} \frac{cdt}{a}$$

For small Δt_e and Δt_0, you find $\dfrac{\Delta t_0}{a(t_0)} = \dfrac{\Delta t_e}{a(t_e)}$

A photon frequency will thus be $\nu_e a(t_e) = \nu_0 a(t_0)$

Luminosity distance

From which it follows, $\frac{a(t_e)}{\lambda_e} = \frac{a(t_0)}{\lambda_0}.$

And using the definition of redshift: $1 + z = a(t_0)/a(t_e)$

Let P be the power emitted by a source at time t and co-moving distance r. Let p_0 be the power per unit area (flux) observed at time t_0. We want Luminosity distance to be such that:

$$p_0 = \frac{P}{4\pi d_L^2}$$

The area of a sphere centered at the source, at time t_0 is $4\pi a_0^2 r_0^2$.

Luminosity distance

Photons emitted by the source are observed with a redshift a/a_0

And photons emitted within a small time Δt_e are observed within a time $\Delta t_0 = a(t)/a_0 \Delta t_e$. Thus:

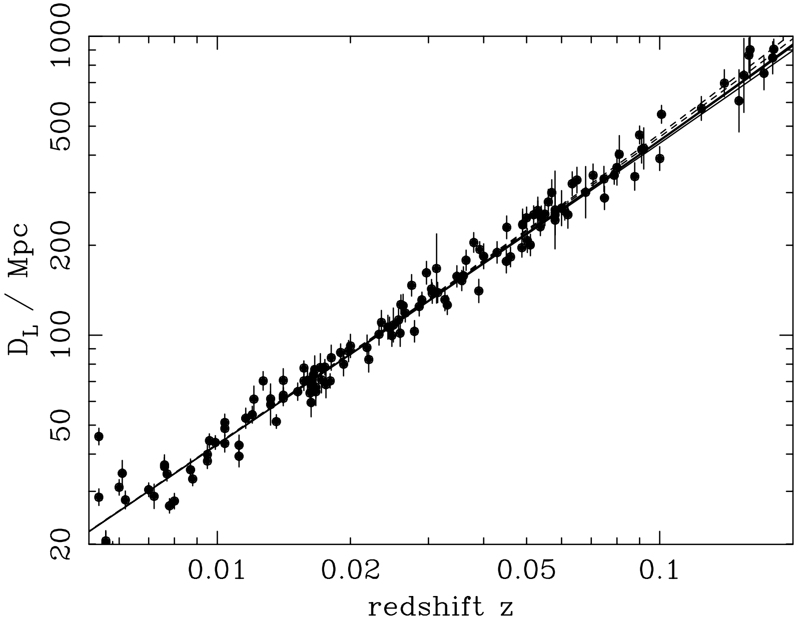
$$p_0 = \frac{P}{4\pi a_0^2 r^2} \left(\frac{a}{a_0} \right)^2 = \frac{P}{4\pi d_L^2}$$

Two factors: one for energy, another for time

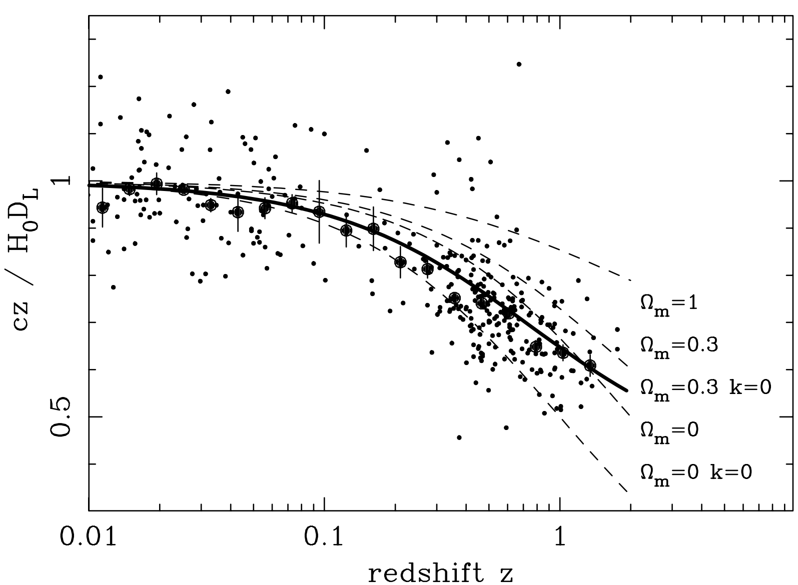
$$d_L = a_0^2 \frac{r}{a}$$

There are other definitions of distance in cosmology – I skip that.

Type Ia Supernovae: Luminosity distance



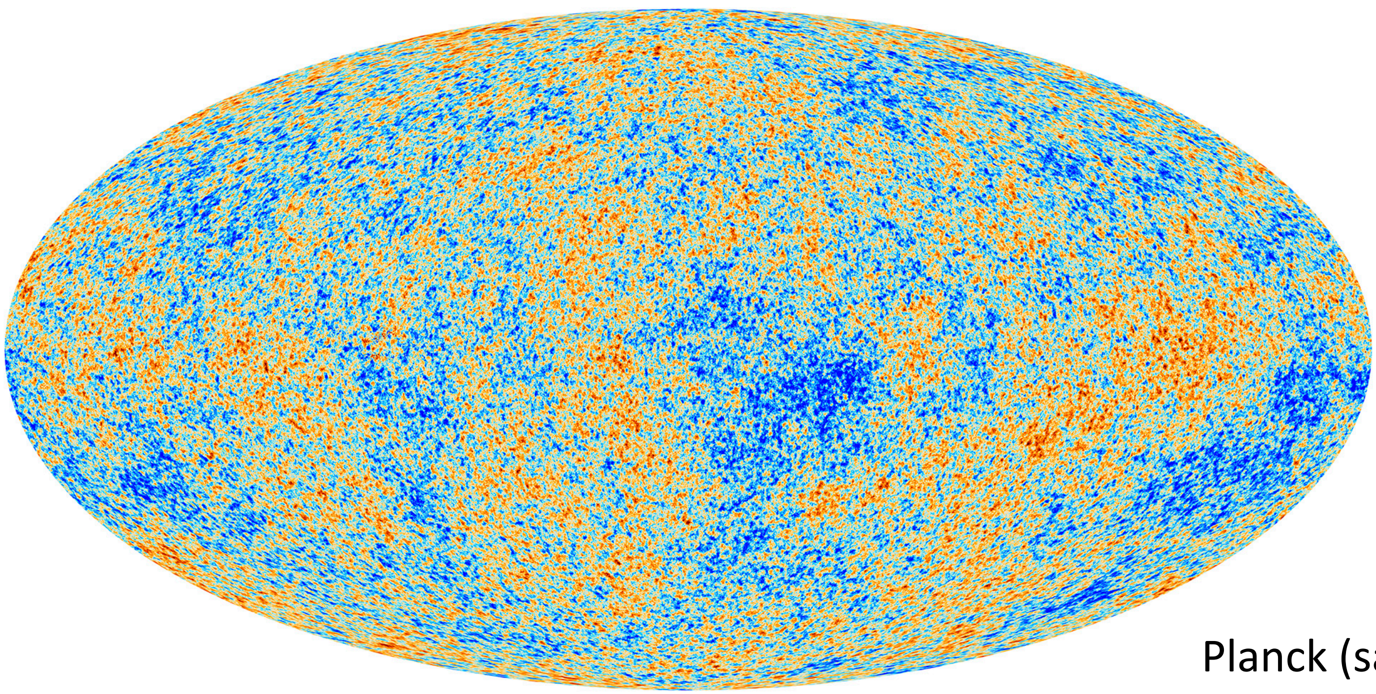
Top plot shows the Hubble diagram obtained with SN type Ia – This confirms that Hubble's law is good and that SN Ia are good standard candles



This is the same plot, but shows. The data is best described for K=0 and Ω_m=0.3. Because K=0 can only be achieved for Ω_{total}=1, this implies that there is a cosmological constant.

Cosmic Microwave background.

It's easy to see that the early Universe, was denser and hotter that it is today. The high density coupled photons to matter. At some past redshift (z~1100) the Universe became transparent to photons. This is era of "last scattering". This background of photons continues to fill the Universe and it has redshifted, due to expansion. It's extremely well described by a Planck function with T = 2.7255 ± 0.0006 K



Planck (satellite)

Density fluctuations

The universe is very homogeneous. But inhomogeneity is observable from individual galaxies to the 100 Mpc scale. The inhomogeneities seen in the CMB grow into today's density inhomogeneities . The density fluctuations are described as:

$$\delta(\vec{x}) = \frac{\rho(\vec{x}) - <\rho>}{<\rho>}$$

It is conventional to describe the density fluctuations as a Fourier expansion (periodic boundary conditions are assumed over a very large volume V).

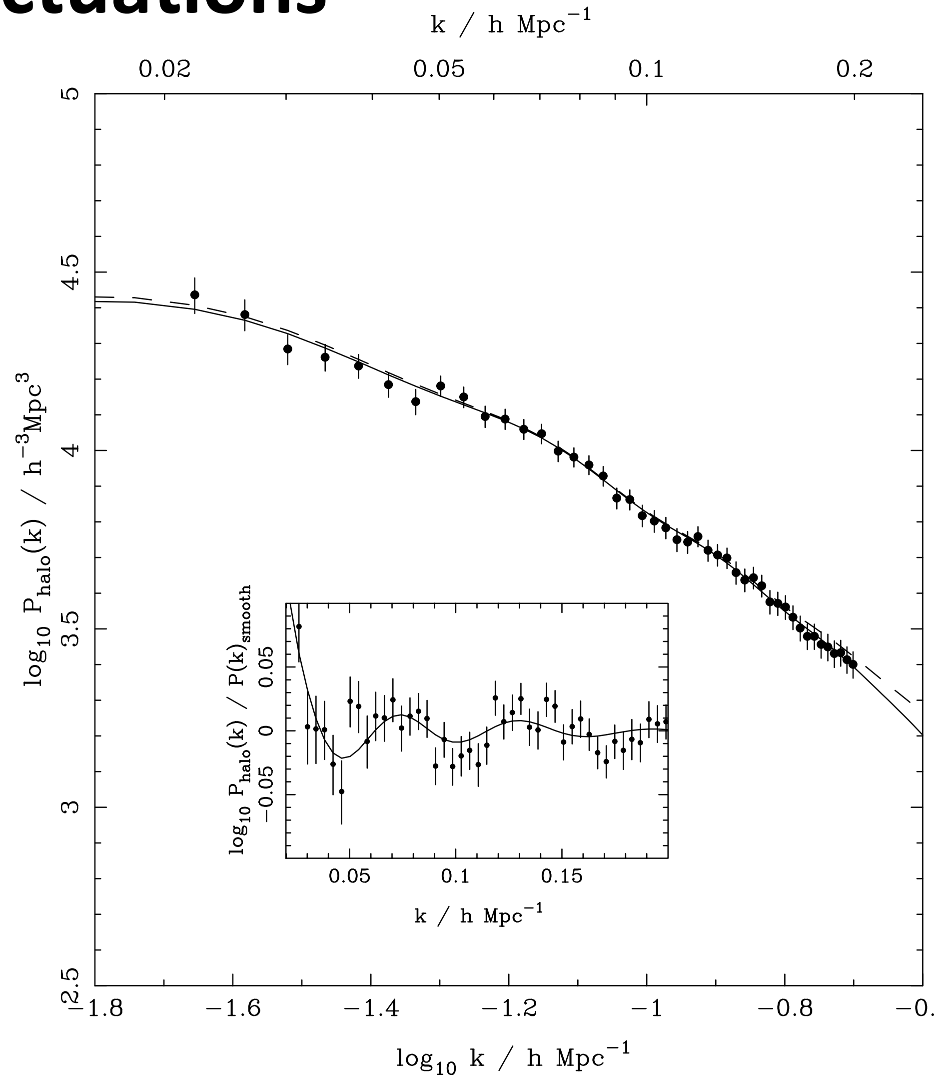
$$\delta(\vec{x}) = \sum \delta_{\vec{k}} e^{-i\vec{k}.\vec{x}}$$

The variance of fluctuations is:

$$< \delta(\vec{x})^2 > = \sum |\delta_{\vec{k}}|^2 = \sum P(k)$$

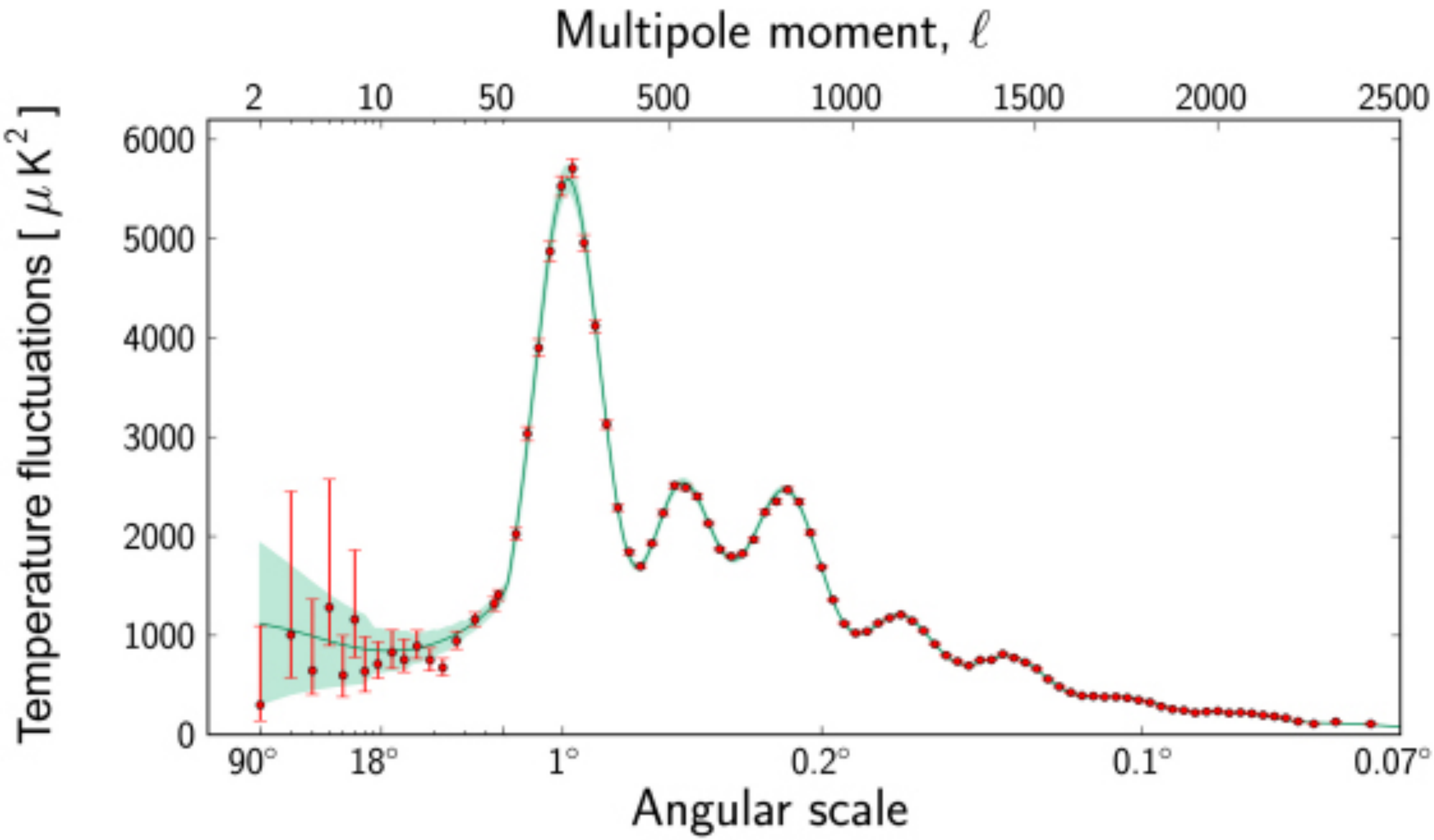
(There no preferred direction for inhomogeneities)

Density fluctuations



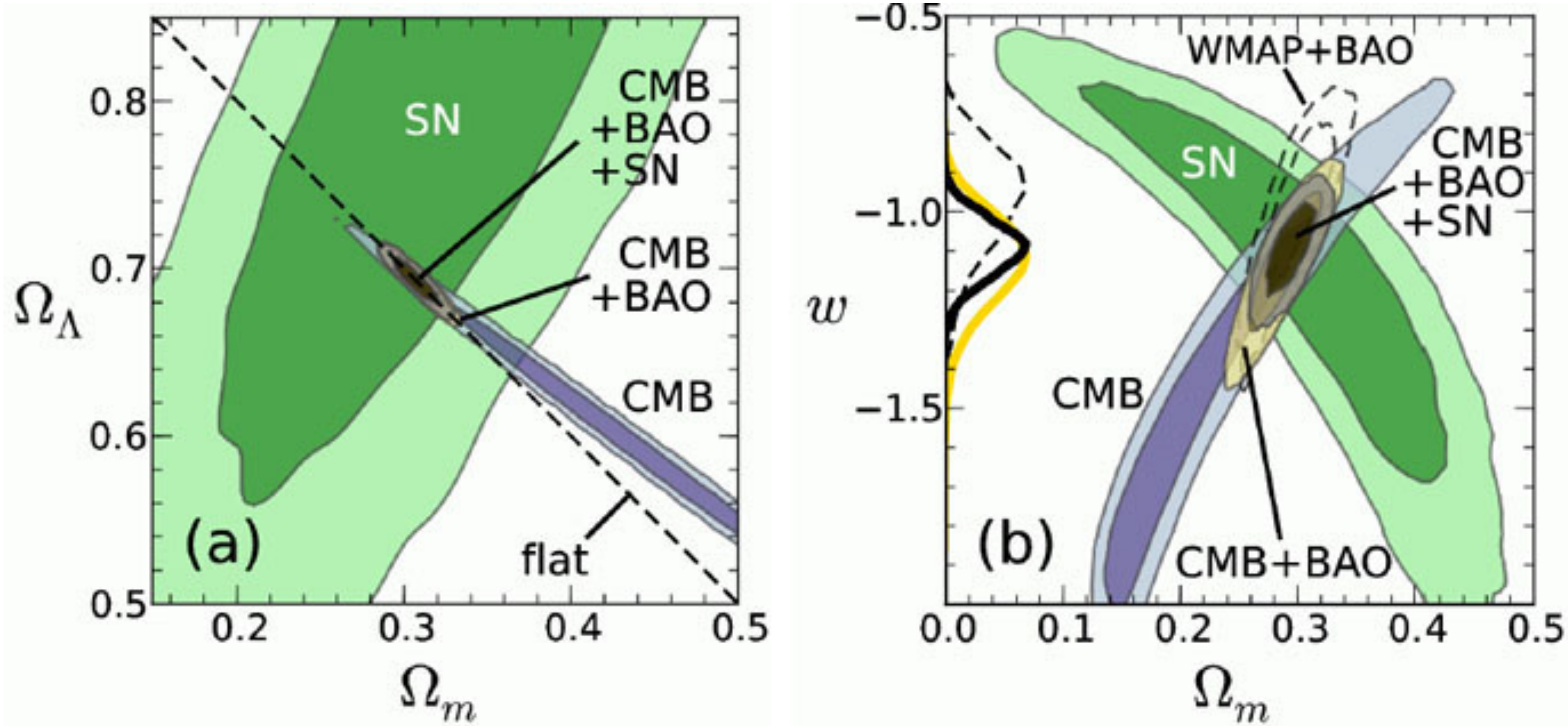
Galaxy power spectrum measured by SDSS. The best fit of ΛCMD is shown. Baryon Acoustic Oscillations are inset.

CMB power spectrum



Power spectrum of the CMB as measured by Planck.

CMB / Sne Type Ia / acoustic baryon oscillations



Best fit region for Planck, SN type Ia data and acoustic baryon oscillations.

Contour lines are for 68.3% and 95.4% confidence levels (1,2 sigma)